Ontological Modularity and Spatial Diversity

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In this paper we propose that adequate treatments of space need to be multiperspectival and related to sound foundational ontologies. To support this, we show that natural spatial descriptions commonly appeal to diverse theories of space and these need to be formally combined to be fully interpreted. Our account draws particularly on the foundational ontology DOLCE and the algebraic specification language CASL. We show how the structuring mechanisms of CASL suggest mechanisms for both building and combining multiperspectival ontologies of space. We also suggest that these mechanisms provide a natural link both with currently emerging cognitive principles such as blending and with developments in ontology mediation and mapping.

Keywords: Ontology, Spatial Ontology, Navigation, Spatial Language, Spatial Semantics, Algebraic Specifications, Modularity, Way-finding

1 Introduction

Much is now known about a rich assortment of spatially-relevant phenomena and frameworks. We have available entire families of formal calculi for qualitative reasoning about space, ranging from treatments of pure regions and their connections (RCC: Randell et al., 1992) to orientation calculi (including Double-cross (Freksa, 1992) and the STAR family of calculi (Renz and Mitra, 2004)), to hybrid systems combining objects of distinct dimensionalities or qualitative and
quantitative information (Egenhofer and Rodriguez, 1999; Gerevini and Renz, 1998). We have formalizations of route knowledge (e.g., Werner et al., 2000; Krieg-Brückner et al., 2005) and some good understandings of the kinds of reference frames that people adopt when describing and, arguably, conceptualizing space (e.g., Levinson, 1996; Eschenbach, 1999). We even have a range of distinct kinds of spatial entities at hand, including relative places (Donnelly, 2005), niches and sites (Smith and Varzi, 1999), anchors (Galton and Hood, 2005), boundaries (Smith and Varzi, 2000) and many more. In short, the problem is fast becoming not one of having formalizations of space, spatial entities and spatial attributes, but far more what to do with the very large number of possible alternatives (and their combinations) that are on offer.

This surfeit of possibilities should be an advantage, as we can pick rather precisely-tuned spatial descriptions as appropriate for the solution of various tasks. We might also beneficially ask questions concerning which aspects of the descriptions find empirical support as cognitively-plausible models and which do not. However current practice is somewhat different. The existence of alternative, perhaps competing, descriptions is in many cases placed on one side while a particular descriptive alternative is explored further. The problem of combining disparate descriptions remains a significant open issue.

In this paper, the issue of combining alternative perspectives on space is our central focus. We argue that it is natural that a broad range of diverse spatial accounts have developed since that is precisely how humans go about dealing with space. By focusing on natural language formulations of certain spatially-embedded tasks, such as route instructions, spatial configuration descriptions and verbalized way-finding activities, for example, we show that it is extremely rare to find an utterance that does not combine contributions from distinct theories of space. Somewhat more far-reaching, we suggest that this is a common feature of language as such: linguistic formulations have as one of their functions the role of combining distinct realms of world-modelling. On the one hand, this relates naturally to notions of conceptual spaces and the prominent role currently being given to the blending of such spaces (Fauconnier and Turner, 2003); on the other hand, it makes direct contact to current efforts in ontological engineering to achieve and manage heterogeneous ontological components where there may be multiple co-existing formalizations that need to be combined.

Our primary goal in this paper will then be to set out how we can capture the kinds of informal theory-combinations concerning space that we see in natural language utterances by employing a properly formalized treatment of ontological modularity. This can move us towards frameworks that provide the benefits and partial solutions of all of the spatial frameworks mentioned above without compromising their intrinsic integrity. We also suggest that formally relating ontological modules is a fundamental component of ontological engineering that goes beyond practical requirements set by, for example, re-use. Indeed, any ontological treat-
ment of an area may turn out to require formally related but distinct ontological modules to be defined simply by virtue of the intrinsic multiperspectivalism of sensible treatments of our everyday reality.

The backbone of our approach is accordingly provided by a solid foundation in ontological engineering. The criteria adopted for deciding when information belongs together and how it is to be articulated draws on ontological considerations, both in terms of the actual categories adopted and in terms of the kinds of methodologies applied during development. Our starting point will be the Descriptive Ontology for Linguistic and Conceptual Engineering (DOLCE: Masolo et al., 2003) and we apply the organizational principles of OntoClean (Guarino and Welty, 2004) throughout. We will demonstrate that a framework of this kind provides a natural home for the distinct sources and organisations of knowledge with which we are concerned.

Our selected area of illustrative application, route descriptions and navigation involving spatial configurations and relationships, provides ample examples of how relations and entities can take on quite different forms, with differing properties and consequences for inference, depending on how they are utilised. In this area we have differences that are due not only to broadly different conceptualizations for various tasks, such as for example, the diverse requirements of representing designed environments from their designer’s perspective vs. their user’s perspectives (cf. Timpf, 2002), but also to the involvement of ontologically distinct realms, such as time, space and physical objects. We aim to explain these divergent ‘properties’ by referring to distinct cleanly defined ontological modules which are then combined as needed.

Formally, we adopt the proposals of Lüttich and Mossakowski (2004) and Krieg-Brückner et al. (2005) and use the logical specification language, CASL (Common Algebraic Specification Language: Mosses, 2004), for ontology specification. In this framework, inter-ontology relationships can be captured as explicit theory morphisms (or certain diagrams of these, see Zimmermann et al. (2006)). This places our work within a growing ontological tradition where category theory is being explored for relating ontologies as theories. This approach has a relatively long tradition in Geographic Information Science approaches to ontology, largely driven by the very clear need to relate diverse geographic perspectives on the world (cf. Frank, 1999, 2001; Fonseca et al., 2002); it is also receiving closer attention in the formal ontology community more generally (cf. Uschold et al., 1998; Krötzsch et al., 2005; Hitzler et al., 2006; Zimmermann et al., 2006). Our principal contributions to this tradition are then: (i) to argue for a thorough axiomatic integration with established foundational ontologies, (ii) to provide a single framework for specifying and structuring relationships between ontological modules that range from simple renamings up to full theory morphisms across components expressed in differing logics, (iii) to illustrate an integrated development environment that enables specifications to be developed, maintained and
verified using standardly available automatic and semi-automatic theorem provers, and 
(iv) to relate the abstract specifications developed to the concrete require-
m ents of applications requiring spatial understanding and spatial dialogue.

We structure the paper as follows. In Section 2, we begin with a brief informal 
motivation of the need to combine ontological modules drawing on linguistic ev-
idence; this draws on a range of established data and empirical experimentation 
that has collectively aimed at revealing how speakers and hearers naturally deal 
with spatial problems of various kinds. Then, having motivated ontological mod-
ularity in broad terms, we set out in Section 3 the ontological foundations for our 
approach and the generic position of space within this. In Section 4, we first show 
how the large-scale structuring techniques of CASL can be used for defining dis-
tinct ontological modules and set out in more detail some treatments of space and 
spatially-relevant categories. We then show how CASL provides sophisticated 
mechanisms for relating modules in ways appropriate for the diversity needed. In 
Section 5 we build on this foundation to present a detailed example in which some 
distinct spatial interpretations must be combined to solve a spatial task. Finally, 
in Section 6, we briefly summarize our results and suggest further directions for 
the future.

2 Ontological diversity in natural utterances

Consider the following rather unexceptional way-finding utterance:

Go from Bremen to Trento, crossing the Italian border at Brennero.

Despite its simplicity, closer examination reveals that a semantically diverse range 
of entities is being invoked. In particular, the first clause, ‘go from Bremen to 
Trento’, is a simple path description, while the second clause, ‘cross the Italian 
border at Brennero’, talks instead of a movement across a boundary between two 
extended spatial regions. Moreover, digging deeper, we can separate out the use 
of the names ‘Bremen’, ‘Trento’ and ‘Brennero’ as identifying points on a path 
description, and ‘Italy’ as naming a geopolitical region. In each case, a range of 
spatial relationships is being brought into play. These relationships can draw from 
the entities explicitly named to a greater or lesser degree. Bremen and Trento as 
named cities are being invoked somewhat minimally in the path description: i.e., 
very little of their physical properties of being extended collections of buildings, 
streets, parks, etc. finds its way into the semantics relevant for understanding the 
example. And, conversely, particular physical objects can find their way into a 
variety of relationships. This is shown in the following collection:

\[
\begin{align*}
\text{This road runs from Bremen to Trento} \\
\text{This road divides Germany from Austria} \\
\text{Go along the road until the church} \\
\text{Go across the road to the church}
\end{align*}
\]
Here some physically identifiable object is used first as a path, or link, between two named entities, second as a boundary between two regions, third as a shape that can be followed parallel to a main axis, and fourth as a shape that can be traversed orthogonally to the main axis; such shifts have received considerable discussion in the linguistic literature on spatial language (cf. Talmy, 2006).

One approach in ontology to capturing what is going on in the above sentences is the direct one of saying, for example, that a road is also a connector and a dividing line and a path that is followed, or that a church is a building with a front and a location and a landmark. But such a move has serious problems in that it confuses distinct entities, each with their own distinctive properties and existence. A conjunctive combination or logical extension of the theory of any of these with the theories of the others is potentially problematic because there is no guarantee, and indeed in general no reason to expect, that the combined theory is self-consistent. An example of this from Geographic Information Systems is where a body of water might be classified according to distinct interest groups as a source of drinking water, a recreational area, a boundary between different land-animal habitats, and a link in a traffic network served by a ferry (cf. Fonseca et al., 2002; Kuhn and Raubal, 2003). The logical properties of a boundary and a link are almost opposite: a boundary divides, a link connects.

The problematic nature of these kinds of potentially contradictory classifications has been handled in a number of ways in the literature, ranging from simple union of diverging properties (e.g., by multiple inheritance) to more sophisticated treatments in terms of roles. For Fonseca et al., for example, roles become a general way of incorporating perspectives or contexts into an ontological account. As they themselves note, however, this involves a substantial extension of the notion of role. A more complete treatment is still required. Here we follow more in the tradition illustrated by Kuhn & Raubal and others, and pursue formalizations of relations between perspectives considered as theories. In particular, we investigate the use of ontological modules and views between such modules as providing a general account of the flexibility of relationships clearly necessary for dealing with space and spatial representations.

3 Formal ontology, axiomatization, DOLCE and space

In the previous section we motivated the need for an ontological system that can consistently manage several types of entities (and their relationships) and that is verifiable. Since our goal is not specific to a particular application and since the main notions we are dealing with (space, object, location) are widely used in (possibly) any domain, we now posit two general requirements: the ontology should be (i) formal, i.e., available as a logical theory (to make possible consistency

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1 We note here that although temporal issues are also involved, particularly in navigation, our discussion here will focus on the spatial aspects almost exclusively due to length constraints.
checks and faithful implementations) and (ii) foundational (to provide an overall framework where, in principle, the disparate entities we need to consider can find room). Among the ontologies in the literature, only a few have been developed addressing (more or less satisfactorily) these requirements. Among these we find DOLCE, the Descriptive Ontology for Linguistic and Cognitive Engineering (Masolo et al., 2003); GFO, the General Formal Ontology (Heller and Herre, 2004); BFO, the Basic Formal Ontology (Smith and Grenon, 2004; Thomas et al., 2004); and OPENCYC (Cycorp, 2004). A comprehensive overview of the individual positions taken in these accounts is given in Farrar and Bateman (2004). Furthermore, from Section 2 we see that an ontological system apt for our task must also allow for different representations and perspectives about space. We thus add a third requirement: (iii) the ontology should not be committed to a specific view of space and spatial qualities. When we consider the third requirement, we find that DOLCE is the only (formal and foundational) system that provides a framework in which alternative space and quality representations can co-exist. We have thus decided to adopt DOLCE as the underlying ontological system for our work.

DOLCE is a first-order axiomatic theory that formalizes a set of quite general categories (top-level), a framework where other categories can be introduced (depending, for instance, on application concerns), and some guidelines on how to formalize new categories. It provides most of what is necessary when attempting to formalize a domain in which spatial representations are required. Here we describe only the part relative to qualities of physical objects since this suffices for our present goals. Although we have all the formal elements necessary to deal with time, for the sake of simplicity in this paper we do not consider temporal changes of physical objects. Thus, we drop temporal parameters in the fragment of DOLCE that we describe. A complete description of the system is given in Masolo et al. (2003).

Physical objects, like a building, a train, a tunnel, a mountain and a forest, form the DOLCE category of physical endurants, i.e., those objects which have a location in space (and time). People differentiate physical endurants because of a variety of aspects and characteristics such as color, weight, smell, size, shape, temperature, etc. Technically, these distinctions are represented in DOLCE via the notions of individual quality, quality-type and quality-space.

Individual qualities (e.g. “the color of my car” and “the weight of John”) inhere in specific individuals, i.e. they existentially depend on specific individuals (my car and John, respectively). The color of my car and, for example, the color of your car are different individual qualities even if both cars happen to be Ferrari-red: the first quality is necessarily related to my car while the latter is only related to your car. In this paper, we focus only on those qualities that are inherent in physical endurants; in DOLCE these are called physical qualities. Individual physical qualities, or simply qualities, are grouped into categories according to the aspects of the objects they represent. These categories are called quality-types: the color
quality-type collects all the individual qualities about color, the weight-quality type all the individual qualities about weight, etc.

Qualities in the same quality-type can be compared according to different quality-spaces that reflect “subjective” (context dependent, qualitative, etc.) points of view. Each quality-space consists of a set of individuals called positions that are organized by means of one or more structural relations (e.g. order, metric, topological, etc.). Individual qualities are then mapped to positions: qualities that share the same position are considered to be indistinguishable. For example, the weight of two amounts of salt may be indistinguishable in the quality-space associated with a baby scale but not in the one associated with a precision balance. In the first case, the two individual weights are mapped to the same position (say 20.10 grams), while in the second case to two distinct weights (say 20.10001 grams and 20.10002 grams). In this sense, positions represent the capacity of a specific quality-space to distinguish individual qualities; by means of different quality-spaces it is possible to choose the needed quantitative/qualitative degrees of similarity, i.e. the required granularities.

The extension of DOLCE presented in Masolo and Borgo (2005) allows each quality-type to be associated with one or more pairwise disjoint quality-spaces (which depend on culture, instruments of investigation etc.). This extension is fundamental for our present discussion. However, in contrast to this extension, we do not require here that an individual quality is necessarily mapped to all the quality-spaces associated with its quality-type. This is important in order to allow for partial information. For example, we might have the position of “the weight of John” in the quality-space associated with a baby scale but not in the one associated with a precision balance. If both positions are given, then the information relative to the same individual quality (and therefore to the same physical endurant) can be combined, i.e. a link between these positions belonging to different quality-spaces can be established “from the bottom”. Alternatively, information in different quality-spaces can be combined “from the top” by systematically linking the positions in different spaces on the basis of external knowledge that we have about the structures of the spaces themselves. Knowing the precision and the scales of the two balances in the previous example, it is possible to introduce a morphism between the associated quality-spaces. In Section 4 we will introduce the formal mechanism we have adopted to express these two ways of combining information, and in Section 5 we will apply them to a specific example.

This approach to qualities and quality-spaces provides a particularly effective set of mechanisms for addressing physical space and spatial information. Differently from other foundational ontologies, DOLCE does not commit to a specific notion of physical space. That is, each physical endurant has a spatial location (an individual quality) that can be mapped to different positions belonging to quality-spaces corresponding to specific views on physical space (e.g. topological, Euclidean, mereological, metric and so on). The spatial location is then considered
as just one among the characteristics of physical endurants. Using this approach to space representation, different ways of structuring the spatial quality-type can be consistently used in the same formalism. Some quality-spaces associated with the spatial location quality-type may be atomic and others atomless; some may satisfy complex mathematical properties and others just basic principles. Spatial operators, like the mereological sum of positions, may be defined in some spaces and not in others, and so on.

We now briefly show how DOLCE (and its extensions) formalizes the notions introduced above.

Formally, we write \( PED(x) \) to mean that \( x \) is in the \( PED \) category, i.e. “\( x \) is a physical endurant”. Similarly, we use \( PQ \) for the category of physical qualities and \( PQ_i \) for the quality-type \( i \) (where \( i \) can be color, weight etc.). Then, expressions \( PQ(x) \) and \( PQ_{\text{color}}(x) \) stand for “\( x \) is a physical quality” and “\( x \) is a color quality”, respectively. The inherence relation between a quality \( x \) and its (unique) physical endurant \( y \) (axioms (A1) and (A2)) is written \( \text{inh}(x, y) \). Physical qualities existentially depend on physical objects (A3) and, given a quality-type \( i \), it is required that an endurant can only have one quality of that type (A4). Among the quality-types we consider at least the spatial location (\( SL \)) and we assume that all physical endurants have an individual quality in \( SL \) (A5).

\[
\begin{align*}
\text{(A1)} & \quad \text{inh}(x, y) \rightarrow PQ(x) \land PED(y) \\
\text{(A2)} & \quad \text{inh}(x, y) \land \text{inh}(x, y') \rightarrow y = y' \\
\text{(A3)} & \quad PQ(x) \rightarrow \exists y(\text{inh}(x, y)) \\
\text{(A4)} & \quad \text{inh}(x, y) \land \text{inh}(x', y) \land PQ_i(x) \land PQ_i(x') \rightarrow x = x' \\
\text{(A5)} & \quad PED(x) \rightarrow \exists y(\text{inh}(y, x) \land SL(y)).
\end{align*}
\]

We write \( \text{pos}(x, y) \) to mean “\( x \) is a position (in a quality-space) of the quality \( y \)” (A6).\(^3\) The set of all physical positions (i.e. positions associated with physical qualities) is indicated with \( PP \) while the set of the positions for a quality-type \( i \) in the quality-space \( j \) is indicated by \( PP_{ij} \).\(^4\) The formula

\[
\text{PQ}_{\text{color}}(y) \land \text{pos}(x, y) \land \text{pos}(x', y) \land PP_{\text{rgb}}(x) \land PP_{\text{cymk}}(x')
\]

tells us that the object having color quality \( y \) has location \( x \) in the RGB color space and has location \( x' \) in the CYMK color space.\(^5\) Each position must live in a single space (A7) and, within a single space, the position is unique (A8).

\(^2\)In the original version of DOLCE this relation is written \( \text{dqt} \).

\(^3\)In the original version of DOLCE this relation is written \( \text{ql} \) and has a temporal argument that it is omitted here because change in time is not being considered.

\(^4\)In the original version of DOLCE physical positions are called physical regions and are written \( PR \). We avoid this usage here in order to achieve a more perspicuous terminological distinction between spatial regions and the abstract formal spaces involved in qualities.

\(^5\)We should note here that there is no mention of qualia in this extension of DOLCE: we leave them aside to simplify the formalization since they do not play a role central to our account here.
As already noted, to allow partial information, we do not require that
\[ \text{pos}(x, y) \to PP(x) \land PQ(y) \]

(A7) if \( j \neq k \) then \( \neg \exists x (PP_j^1(x) \land PP_k^1(x)) \)

(A8) \( \text{pos}(x, y) \land \text{pos}(x', y) \land PP_j^i(x) \land PP_j^i(x') \to x = x' \).

The weaker position
\[ \text{PQ}_i(x) \to \bigvee_j (\exists y (\text{pos}(y, x) \land PP_j^i(y))) \]

could be more acceptable, but we prefer an even weaker commitment accepting qualities without any position, i.e., we just strengthen (A6) assuming:
\[ \text{PQ}_i(x) \land \text{pos}(y, x) \to \bigvee_j (PP_j^i(y)). \]

The general framework introduced is illustrated in Figure 1. Note that inh is an injective function and that pos is a (partial) function.

![Diagram](image)

**Figure 1.** The DOLCE mechanism of quality-type and quality-spaces.

Considered formally it might appear that qualities can be avoided by introducing (partial) functions, one for each quality-space, that directly link physical endurants to positions as suggested in Figure 2. As discussed in Masolo and Borgo (2005), however, it would then be impossible to express the fact that, for example, the quality-spaces with positions in \( PP_1^1, \ldots, PP_{m_1}^1 \) describe different points of view on the *same* aspect of physical endurants, like the RGB and the CYMK color spaces. In contrast, the quality-spaces with positions in \( PP_1^n \) and \( PP_{m_n}^n \) describe different characteristics of endurants, like color and weight for example.
By means of quality-types, it is possible in DOLCE to “cluster” all the quality-spaces relative to the same aspect. An equivalence relation between positions could simulate this clustering, but we prefer the former approach because it provides an ontological “interpretation/explanation” of the clustering. In addition, we can note that it is possible only in our approach to represent the fact that a physical endurant has a quality of a specific type (for example, that it has a color-quality) even though no positions of that quality are provided in any quality-space. This provides a mechanism to represent very general properties of endurants (for example the fact that they are colored, independently of which color they have) that can be useful in applications.

![Figure 2: Avoiding qualities.](image)

Taking all these issues into account, in the following sections we explore how to use quality-spaces and to relate them for the application considered in this paper. The general goal is to combine information that is captured in two (or more) different spaces about the same quality-type or that comes from spaces about different quality-types.

### 4 Large-scale structuring for ontological modules with CASL

Having introduced our ontological position and the possibility that DOLCE offers for capturing multiple perspectives on space, we show in this section how this can be formally structured in terms of ontological modules defined using the Common Algebraic Specification Language, CASL (Mosses, 2004). To do this, we first introduce the basic language constructs relevant for richly axiomatized ontologies as originally proposed by Lüttich and Mossakowski (2004) and then go on to show
how the basic CASL language construct of the view supports the specification of theory morphisms that bring different ontological modules into relation with one another.

4.1 Ontology specifications and modules

CASL is described in detail in, for example, Mosses (2004); here we focus particularly on the use of CASL for ontology specification and proceed by means of a brief example. In the following specification taken from our current work involving formalization of the DOLCE ontology as a CASL specification, we can see the basic structuring mechanisms provided. This specification constructs the axiomatization of qualities introduced in the previous section and contains component specifications, bracketed within spec . . end keywords. Each CASL specification can be seen as a logical theory.

spec QUALITYType[sorts q, Elem] =
  pred inh : q × Elem
  ∀ x, x': q; y, y': Elem
  • inh(x, y) ∧ inh(x, y') ⇒ y = y'
  • inh(x, y) ∧ inh(x', y) ⇒ x = x'
  • ∃ y : Elem • inh(x, y)
end

spec STRONGQUALITYType[sorts q, Elem] =
  QUALITYType[sorts q, Elem]
then ∀ y : Elem • ∃ x : q • inh(x, y)
end

spec QUALITYSpace[sorts s, q] =
  pred pos : s × q
  ∀ x, x': s; y : q • pos(x, y) ∧ pos(x', y) ⇒ x = x'
end

spec PARTHOOD =
  PARTIALORDER with _<=_=→ P
dend

The first specification, QUALITYType, sets up the basic property of qualities of being unique to the entities that they inhere in by stating appropriate axioms on the predicate inhere (inh). We also declare at this point some sorts (types), which are interpreted loosely as any sets, and the typing of the predicate. The correct typing of each predicate in a CASL specification is statically checked by the CASL development environment, Hets (Mossakowski et al., 2007). Clearly further basic
properties of the inh relation can be axiomatized here as required. QUALITY-TYPE is extended (extension is marked by the keyword then) to the specification STRONGQUALITYTYPE. These quality types are qualities that are required to inhere in particular entities. This allows us later to state that each physical endurant necessarily has a spatial location. The third specification QUALITYSPACE defines the position relation (pos). Finally, we use renaming (written using the keyword with) to define the usual ontological mereological notion of PARTHOD on the basis of the standard CASL theory PARTIALORDER: the \( \leq \) relation of the partial order is renamed as the \( P \) of parthood; we also show the definition of PARTIALORDER below when we discuss theory morphisms. Renaming of specifications, in particular in combination with their union (written using the keyword and) allows specifications to be re-used and avoids their repetition (as demonstrated by the specification APPLICATIONQUALITIES in Section 5 below).

Combining specifications in this way corresponds to a strong notion of ontological modularity. Our specification of DOLCE consists of many such modules, expressed in terms of individual specifications/theories. These are then combined in a single composite specification via union. This kind of modularity is of considerable benefit when working with large axiomatized ontologies since it allows consistency and other properties to be checked on the smallest possible extracts of the ontology before proceeding to their combination. Ontological modules of this nature are therefore components of a single combined ontology and so can also be termed sub-ontologies. Naturally, such modules can be re-used to build other ontologies and provide a formalized and implemented rendering of the well-known proposal to consider ontologies in terms of a ‘lattice of theories’ as promulgated by John Sowa and others.

4.2 Relations between modules as theory morphisms

Constructing composite specifications by means of union and extension is not yet sufficient for capturing the account of space set out within DOLCE in the previous section. Here there is a different kind of modularity at work. We have seen that DOLCE supports the formalization of alternative perspectives on space and other qualities. In general, these perspectives may draw on quite different characterizations that would resist simple composition. To formalize these notions, each space \( S_i \) described in Section 3 above is nevertheless formalized as a, possibly composite, CASL theory. However, particularly in the context of the SFB/TR8 Collaborative Research Center on Spatial Cognition, we now have access to a broadening range of spatial calculi receiving a formalization in CASL. These vary across Doublecross, RCC5, RCC8, Route Graphs, and several others (cf. Wöllfl and Mossakowski, 2005; Krieg-Brückner and Shi, 2006). Although we may still refer to these theories as sub-ontologies, in one particular sense they can also be seen as alternative ontologies. Each such theory corresponds to a particular candi-
date characterization of the world and so each such theory represents a candidate ‘ontology’ of space.

This directly relates the mechanisms that we employ for relating such alternatives to the broader case of considering relations between distinct ontologies in general. The definition and inter-relation of ontologies as such is currently being seen as a crucial problem for the development of ontologically-based semantic treatments, such as, for example, for the semantic web and its inescapable heterogeneity in contents, forms and intentions. Its relevance for relating distinct kinds of spatially-relevant information has also recently been argued in terms of semantic reference systems by Kuhn and Raubal (2003). The issue itself is an old one, however, going back to the earliest attempts to provide re-usable ontological components for ontological engineering. Current work within the ontology community focusing on sophisticated re-use is moving to richer and more formalised frameworks, such as the Information Flow Framework developed mathematically by Barwise and Seligman (1997) and promoted for ontology use by Kent (2000). There are now an increasing number of proponents of this and similarly strong mathematical accounts for the problem of combining and re-using ontologies. Our own approach therefore falls within this new ‘tradition’.

The essential construct provided by CASL for supporting this aspect of our ontology work is the view. Views provide a means for defining a logical interpretation of a source theory in terms of a target theory. The constraint on their definition is that the axioms of the source specification, when translated along the view, must be provable in the target specification. This amounts to an interpretation of theories in the sense of logic.

Establishing views between specifications allows for theories to be proved that bind specifications together in general. Thus, any knowledge that we have about one domain is then generally applicable to the other. The following shows this in action and also sets up some of the constructs we will use in our example below. We first provide some quite generic specifications concerning basic relations with wide application: the mathematical notions of pre-orders and partial orders. We then also provide a specification for the notion of connection as used in connection calculi and other treatments of space (for more details of this, see, for example, Casati and Varzi, 1999).

```
spec PREORDER =
  sort Elem
  pred _ <= _ : Elem x Elem
  \forall x, y, z : Elem
  • x <= x                  % (refl)%
  • x <= z if x <= y \& y <= z % (trans)%
end

spec PARTIALORDER =
```
PreOrder
then $\forall x, y : \text{Elem} \bullet x = y$ if $x \leq y \land y \leq x$ \hspace{1cm} \%(\text{antisym})$
end

spec \ CONNECTION =
sort $\text{Elem}$
pred $\_\cdot\_\text{connected}\_ : \text{Elem} \times \text{Elem}$
\forall r, s : \text{Elem}
\bullet r \text{ connected } r \hspace{1cm} \%(\text{refl\_connected})$
\bullet s \text{ connected } r \Rightarrow r \text{ connected } s \hspace{1cm} \%(\text{sym\_connected})$
\bullet s = r \Leftrightarrow \forall z : \text{Elem} \bullet z \text{ connected } r \Leftrightarrow z \text{ connected } s \hspace{1cm} \%(\text{equal\_regions})$
end

We can then establish a general relationship between the theory of connection and the theory of partial orders by providing and then proving the following view.
view \ CONNECTION\_INDUCES\_PARTIAL\_ORDER :
\{ \text{PartialOrder} to \}
\{ \text{Connection}
then pred $\_\leq\_ : \_\cdot\_\text{, Elem} \rightleftharpoons$
\forall z : \text{Elem} \bullet z \text{ connected } x \Rightarrow z \text{ connected } y \hspace{1cm} \%(\leq \text{ def})$
\}
end

Given this we know that we can move between the theory of connection and the theory of partial order at will: results proved of one can be used of the other. The proof statuses shown in Figure 3 and explained below demonstrate the view as a whole has been proved and can be relied upon in subsequent specifications.

Work with CASL specifications proceeds via the development tool Hets (Mossakowski et al., 2007). The primary user interface of Hets is formed by the \textit{Structure Graph}, which shows dependencies between specifications and the proof statuses of their component theories. In Figure 3 we see an extract of the Structure Graph of the specifications for our examples in this paper. Each named node represents the final theory of a structured specification and each unnamed node represents intermediate steps.\textsuperscript{6} The unnamed node below \textit{CONNECTION} in the graph, for example, is the extension of \textit{CONNECTION} with the predicate '$\_\leq\_' defined in terms of \textit{connected} as needed for the view \textit{CONNECTION\_INDUCES\_PARTIAL\_ORDER} we have just given.

Arrows always point in the direction of development; here there are two different types shown: (1) Dark (black) arrows denote the usual extension and union of specifications; and (2) light grey (green) arrows denote views. Some of the grey arrows are introduced on the fly for the instantiation of generic specifications like \textit{DISTANCE} (defined below) and those leading to the grey (red) node

\textsuperscript{6}Views may also have intermediate specifications.
below CONNECTION and PARTIAL ORDER. The arrows and nodes indicate proof statuses and outstanding proof obligations. These can be examined directly by activating associated inspectors. Some inspectors for edges, showing signature morphisms, and for nodes, displaying theories—both derived automatically from the specification—are also shown in Figure 3. The thick arrows with open heads connect the opened inspection windows to their corresponding nodes or arrows.

Hets allows for the automatic decomposition of views into theorems. Some of them can be proved on the structured level, which Hets can perform automatically, while others have to be shown with some automatic or semi-automatic theorem prover executed by Hets. The window shown in the lower right of the figure is used for the delegation of goals to the chosen prover. In the figure as displayed, all the selected goals have just been proved (in this case by the standard theorem prover SPASS) as shown by the [+] in front of the goal names. This means that the proof obligations for the view have been discharged: pre-order and partial order have been shown in terms of connection and so the view holds. After achieving a proof, Hets shows and stores the proofs in a uniform way so that they can be reused in subsequent development. In this way, it is possible to build up composite
theories in which diverse spatial perspectives have been formally combined as
required but without compromising the ontological modularity of the components.

We can then move on to provide further details more specific to space and
which will be used in our example below. First, we provide a specification of
navigation networks defined on the basis of route graphs. Krieg-Brückner et al.
(2005) have already provided a generic specification of route graphs; this has
been augmented further by an axiomatization of orientation calculi, particularly
Doublecross, by Krieg-Brückner and Shi (2006). For present purposes, we focus
only on the linked graph-like aspect of the representation by defining route graphs
to be a specialization of linked lists. This allows a path to be defined as a particular
kind of linked list from one node to another as follows:

\[
\text{spec RouteGraph} = \\
\text{LinkedList} \text{[sort node} \\
\text{pred link : node } \times \text{ node]} \text{]

then %def
\text{pred path : node } \times \text{ node}
\forall x, y : \text{node}
\bullet \text{path}(x, x) \quad \%\text{(refl path)\%}
\bullet \text{path}(x, y) \Leftrightarrow \exists L : \text{List[node]} \bullet \text{LinkedList}(x :: (L ++ [ y ])) \quad \%\text{(path def)\%}
end
\]

LinkedList is itself defined in the usual way that lists are specified and so will
not be spelled out explicitly here. Relevant is only the use of a parameterized
specification, shown by the square brackets, that makes available its nodes and
links for the subsequent definition of paths. We then relate links and paths as
such, and without reference to other theories, as follows:

\[
\text{spec WeakRouteGraph} = \\
\text{sort node} \quad \text{preds link, path : node } \times \text{ node} \\
\forall x, y, z : \text{node}
\bullet \text{path}(x, x) \quad \%\text{(path refl)\%}
\bullet \text{link}(x, y) \Rightarrow \text{path}(x, y) \quad \%\text{(links are paths)\%}
\bullet \text{link}(x, y) \land \text{path}(y, z) \Rightarrow \text{path}(x, z) \quad \%\text{(link prefix of path yields path)\%}
end
\]

This structured and modular definition of route graphs (which could then be
augmented further as required, for example by relating to particular spatial calculi)
also allows a further simple example of the utility of expressing views between
theories. First, we combine our specification of route graphs based on linked lists
and the weaker specification\(^7\) that relates links to paths:

\(^7\)It is weaker because it does not preclude e.g. that the path predicate holds everywhere.
view PathApproximation :
  WeakRouteGraph to RouteGraph
end

This (proved) view establishes that the two specifications are in important respects interchangeable. We can then use the weak route graph for theory proving when the more complex specification is not required. This kind of modularity is used to good effect below since it pre-structures and simplifies the sets of axioms that need to be considered when proving theorems automatically.

Then, we establish a result concerning properties of route graphs in general by specifying and proving a view between route graphs and pre-orders:

view path_on_RouteGraph_is_a_PreOrder :
  PreOrder to RouteGraph = __<=__,__ -> path
end

This informs us that all theorems provable with respect to the pre-order are also provable with respect to path. We can subsequently rely on this for development and specification in that any attempt to construct a route graph that does not uphold this property will not be consistent with what has been established this far. Moreover, we can employ such general constraints to aid in combining information sources: in order to maintain the overall consistency of a route graph, it may be necessary, for example, to combine certain nodes or to posit the existence of undiscovered nodes. Finally, we employ this view in the next section for simple way-finding.

5 Putting it all together (while keeping it all apart)

In this section, we present a single running example that involves a set of route instructions that necessarily involves combinations of information from various perspectives. Our main aim is to show how a modularized ontological framework can cover the necessary information without in any way compromising the modelling of the individual areas. The relations constructed between areas of ontological specification, which, as we described above, may be regarded either as ontology submodules or alternative ontologies depending on the particular area addressed, are of several different kinds. At the most basic level, specifications will be brought together by relating particular instances; slightly more abstract are relations between specifications brought about either by renamings of relations and entities or by unions of modules; and most abstract still are the general relations between alternative perspectives achieved by specifying morphisms between theories. The least abstract hold for particular elements that we are making statements about; the most abstract provide theories about how distinct areas of specification necessarily correspond and so hold for all elements that may be described using the theories brought into relationship with one another.
The example is as follows. It is seen as a possible answer to be generated by an information system when asked by its user to provide a way-finding description to a branch of the user’s bank. It is our goal to provide a sufficiently detailed and ontologically motivated specification of the situation so as to support the generation of such responses automatically.

The closest branch of your bank is in Chinatown. Take the subway here and get off at Tung Station. It’s the only station in that area. There are three banks that are not so far from the station. I suggest you to go to the one behind Jiang Square. You can recognize it easily because it is a red building.

The example is constructed in order to bring out a reasonable range of complexities but without requiring explanations that would be beyond the detail possible in this paper. However, based on our analyses of a variety of route instruction corpora, we can state that the example is not particularly odd concerning the combination of information that it requires. There is precisely the mix of spatial information of different kinds as well as other physical qualities that supports landmark identification.

The spatial situation that is assumed for our example is shown in Figure 4. Here we see the assumed starting point (the subway station ‘Central Station’) linked via a navigation network to the target subway station (‘Tung Station’) that lies within the city administrative region ‘Chinatown’. In addition to the banks, which are shown with a partially discriminating physical quality of color, there is...
an alternative route via a fountain (marked by ‘F’) — this is in the world but was not used in the description.

A location quality space defines a particular perspective on space, i.e., a particular range of possible positions for the qualities involved. In a region-based space, the possible positions are given by regions, their connections and subregions; in a route-graph based space, the possible positions are given by nodes in the route network. Physical objects are therefore positioned with a range of diverse spatial perspectives. Often, relating between these is not possible ‘directly’ and composite mappings from one space to another may need to be constructed. A central feature of our account will be how we use physical endurants (e.g., a bank or a square) to bring together positions within distinct quality spaces.

Our description of the example will proceed as follows. We set out the specification that is required to capture the relevant aspects of the world — these are formed from a combination of generic foundational information imported from our ontological basis and particular entities specific to the described domain. On the basis of this specification, we will then show that the way-finding description to be generated can be proved as a theorem following from the specification using the support tools introduced in the previous section.

We can start by setting up the notion of an administrative area in a city as follows. A CITYAREA is defined by using CONNECTION from above; the elements that may be connected then yield CITYAREAS. A specification CITYAREA_PARTHOOD then combines information from both CITYAREA and PARTHOOD, again with appropriate import and renaming of the required entities by means of parameterization.

\[
\text{spec } \text{CITYAREA } = \\
\text{CONNECTION with } \text{Elem } \mapsto \text{CityArea}
\]

\[
\text{end}
\]

\[
\text{spec } \text{CITYAREA\_PARTHOOD [PARTHOOD with } \text{Elem } \mapsto \text{CityArea}[k1]] \\
\quad [\text{CITYAREA with } \text{CityArea } \mapsto \text{CityArea}[k2]] =
\]

\[
\text{free type Areas } ::= \text{sorts } \text{CityArea}[k1], \text{CityArea}[k2]
\]

\[
\text{and } \text{PARTHOOD with } \text{Elem } \mapsto \text{Areas}, \text{P } \mapsto \text{isIn}
\]

\[
\text{end}
\]

A free type declaration imposes a fixed (non-loose) interpretation to a sort; here, Areas is declared to be the disjoint union of CityArea[k1] and CityArea[k2]. Following these specifications we can talk of (and prove theorems about) city areas as being in other city areas (isIn) and as being connected.

We also require a way of talking about qualitative distance for our example. This further perspective on space we introduce very simply as follows; again, it is to be emphasized that this can be further complicated or augmented as required.

\[
\text{spec } \text{DISTANCE[sort Elem] =}
\]
Here, the free type declares an enumeration type consisting of exactly four elements. distance is declared to be a commutative binary operation. And, to permit us to talk about distance between city areas, we combine these into a MAP specification thus:

```
spec MAP =
  DISTANCE[CITYAREA fit sort Elem ↦ CityArea]
end
```

The next stage is to specify the basic ‘ground facts’ of the example. In general this is a relatively simple process because it draws directly on the foundational ontology and the categories that it provides. Most relevant for us here is the situation when we have available descriptions that rely on distinct quality spaces. As we saw in Section 2 above, natural language descriptions can draw on a diverse range of kinds of space—we can take this flexibility over into our modelling directly. Thus the following specification states that we have the strong quality types of color and spatial location. We have one kind of color quality space (RGB) and several spatial quality spaces: two that are instantiations of kinds of route graphs—one for the subway stations and one for the city entities; and two that are built on city areas—one for buildings and the other for administrative areas. The basic entities of the domain are also defined at this point: stations and banks are buildings, buildings are physical endurants, etc. The remaining parts of the specification—for example, stating that physical endurants (PED) are subtypes of endurants (ED), etc.—are given by DOLCE; we include them here to make the example locally complete because we use precisely these specifications in the automatic proofs reported on below.

```
spec APPLICATIONQUALITIES =
  STRONGQUALITYTYPE [sorts Color,PED fit q ↦ Color, Elem ↦ PED]
and QUALITYSPACE [sorts RGB, Color fit q ↦ Color, s ↦ RGB]
and STRONGQUALITYTYPE [sorts SL, PED fit q ↦ SL, Elem ↦ PED]
and QUALITYSPACE [sorts node[sub], SL fit q ↦ SL, s ↦ node[sub]]
and QUALITYSPACE [sorts node[city], SL fit q ↦ SL, s ↦ node[city]]
and QUALITYSPACE [sorts CityArea[Building], SL
  fit q ↦ SL, s ↦ CityArea[Building]]
and QUALITYSPACE [sorts CityArea[Area], SL
  fit q ↦ SL, s ↦ CityArea[Area]]
then sorts Station,Bank < Building; Building, Square, Fountain < PED;
```
Within this overall structure we now identify the particular entities involved and their properties. Particular nodes are located in particular types of route graphs, particular spatial locations are identified via position in spatial quality spaces, particular colors are given via position in color quality spaces, etc. The individual qualities and relations necessary for our example are given in the following specifications; their connection with the entities in which they inhere follows below. The reader can readily confirm that the information contained here is essentially local and could be provided straightforwardly for the situation depicted graphically in Figure 4 above. The appended parts of the variable names (e.g., sl in B1 sl) are a mnemonic device to show the kind of information involved (e.g., the spatial location of building 1): this, of course, plays no role for the formal properties of the specifications defined. Nodes in, for example, a route-graph derived specification only receive positions within a route-graph derived quality space, and so on.

```
spec SubwayRG_of_Domain =
  QUALITYSpace [WeakRouteGraph then sort SL fit q → SL, s → node]
then ops
  Central_Station ns, s1 ns, s2 ns, TungStation ns, s4 ns : node;
  Central_Station sl, s1 sl, s2 sl, TungStation sl, s4 sl : SL
• link(s1 ns, Central_Station ns)
• link(Central_Station ns, s2 ns)
• link(Central_Station ns, TungStation ns)
• link(s2 ns, TungStation ns)
• link(TungStation ns, s4 ns)
• pos(Central_Station ns, Central_Station sl)
• pos(s1 ns, s1 sl)
• pos(s2 ns, s2 sl)
• pos(TungStation ns, TungStation sl)
• pos(s4 ns, s4 sl)
end
```

```
spec ChinatownRG_of_Domain =
  QUALITYSpace [WeakRouteGraph then sort SL fit q → SL, s → node]
then ops
  TungStation nc, Sq nc, B1 nc, B2 nc, B3 nc : node;
  TungStation sl, B1 sl, B2 sl, B3 sl, Sq sl : SL
```
• link(TungStation(nc), Sq(nc))
• link(Sq(nc), B1(nc))
• link(TungStation(nc), B2(nc))
• link(TungStation(nc), B3(nc))
• pos(TungStation(nc), TungStation(sl))
• pos(B1(nc), B1(sl))
• pos(B2(nc), B2(sl))
• pos(B3(nc), B3(sl))
• pos(Sq(nc), Sq(sl))
end

spec Map_of_Domain =
QUALITYSPACE[Map then sort SL fit q → SL, s → CityArea]
then ops B1_sl, B2_sl, B3_sl, s2_sl, TungStation_sl, s4_sl : SL;
   B1_creg,B2_creg,B3_creg, TungStation_creg,s2_creg, s4_creg:CityArea
   • pos(s2_creg, s2_sl)
   • pos(TungStation_creg, TungStation_sl)
   • pos(s4_creg, s4_sl)
   • pos(B1_creg, B1_sl)
   • pos(B2_creg, B2_sl)
   • pos(B3_creg, B3_sl)
   • distance(B1_creg, TungStation_creg) = not so far
   • distance(B2_creg, TungStation_creg) = not so far
   • distance(B3_creg, TungStation_creg) = not so far
end

spec Colors_of_Domain =
QUALITYSPACE[sorts Color, RGB fit q → Color, s → RGB]
then ops B1_co, B2_co, B3_co : Color
free type RGB ::= red | green | blue | grey
• pos(red, B1_co)
• pos(red, B3_co)
• pos(green, B2_co)
end

spec Areas_of_Domain =
CityArea
then ops Chinatown, NotChinatown : CityArea
• ¬ Chinatown = NotChinatown  %(Chinatown is distinct from NotChinatown)%
• Chinatown connected NotChinatown  %(Chinatown con. NotChinatown)%
• ∃ x : CityArea • ¬ x = Chinatown ∧ ¬ x = NotChinatown
   %(existence of an area apart from Chinatown and NotChinatown)% %implied
We bind together particular elements of the disparate perspectives in several ways. For route-graph derived spaces we specify transitions that provide a simple connection between route graphs (analogous to the transfers described between route graphs in Krieg-Brückner et al. (2005)).

```
spec RouteGraphTransition [RouteGraph with node ↦ node[1]]
    [RouteGraph with node ↦ node[2]] =
    free type nodes ::= sorts node[1], node[2]
and RouteGraph with node ↦ nodes
end
```

This combines the notion of Tung Station as a node in a subway network and Tung Station as a node in a network of navigation paths around the city (presumably via roads, walkways, etc.). Note that at this point in the specification, we do not know anything further that relates the two entities.

```
We combine the areas that buildings occupy in the city with their respective administrative areas with the following specifications. First we establish a morphism between the city area map and parthood in order to reason about the administrative areas and to specify isIn relations. This is very similar to the view relating parthood and connection illustrated in Section 4 above. Then we relate city regions identified by buildings with their containing administrative areas.
```

```
view Map_of_Domain_induces_Parthood :
    {Parthood with Elem ↦ CityArea[kl]} to
    {Map_of_Domain with CityArea ↦ CityArea[Building]}
```
\[
\text{then pred } P(x, y : \text{CityArea[Building]}) \iff \\
\forall z : \text{CityArea[Building]} \bullet z \text{ connected } x \Rightarrow z \text{ connected } y \quad \%(P_{\text{def}})\%
\]

\[
\text{end}
\]

\[
\text{spec BigChinatown} = \\
\text{CityArea Parthood [view Map_of_Domain_induces_Parthood]} \\
[\text{Areas_of_Domain} \\
\text{with CityArea }\Rightarrow \text{CityArea/Area}] \\
\text{then} \bullet \text{isIn}(B1c_{\text{reg}}, \text{Chinatown}) \\
\bullet \text{isIn}(B2c_{\text{reg}}, \text{Chinatown}) \\
\bullet \text{isIn}(B3c_{\text{reg}}, \text{Chinatown}) \\
\bullet \text{isIn}(\text{TungStation}_{c\text{reg}}, \text{Chinatown}) \\
\bullet \text{isIn}(s2c_{\text{reg}}, \text{NotChinatown}) \\
\bullet \text{isIn}(s4c_{\text{reg}}, \text{NotChinatown}) \\
\text{end}
\]

With these specifications complete, it only remains to combine them all into a complete characterization of the domain. This is achieved by the following union of specifications and the given extensions:

\[
\text{spec BuildingsGraphsAndColors_of_Domain} = \\
\text{Colors_of_Domain} \\
\text{and ApplicationQualities} \\
\text{and Subway_connected_Chinatown_of_Domain} \\
\text{and BigChinatown} \\
\text{then sorts Areas, nodes } < S \\
\text{ops } B1, B2, B3 : \text{Building}; \\
\text{Central_Station, s1, s2, TungStation, s4 : Station}; \\
\text{Sq : Square} \\
\text{preds PATHSL}(sl1, sl2 : SL) \iff \\
\exists b1n, b2n : \text{nodes} \\
\bullet \text{pos}(b1n, sl1) \land \text{pos}(b2n, sl2) \land \text{path}(b1n, b2n); \quad \%(\text{PATHSL def})\% \\
\text{PATH_PED}(b1, b2 : \text{PED}) \iff \\
\exists b1sl, b2sl : \text{SL} \\
\bullet \text{inh}(b1sl, b1) \land \text{inh}(b2sl, b2) \land \text{PATH_SL}(b1sl, b2sl) \\
\%(\text{PATH_PED def})\% \\
\bullet \text{inh}(\text{Central_Station}_sl, \text{Central_Station}) \\
\bullet \text{inh}(s1sl, s1) \\
\bullet \text{inh}(s2sl, s2) \\
\bullet \text{inh}(\text{TungStation}_sl, \text{TungStation}) \\
\bullet \text{inh}(s4sl, s4) \\
\bullet \text{inh}(\text{Sq}_sl, \text{Sq}) \quad \%(\text{inh Sq})\% \\
\bullet \text{inh}(\text{B1}_sl, \text{B1}) \quad \%(\text{inh B1})\%
• \( inh(B2_{sl}, B2) \)
• \( inh(B3_{sl}, B3) \)
• \( inh(B1_{co}, B1) \)
• \( inh(B2_{co}, B2) \)
• \( inh(B3_{co}, B3) \)

By these means, the entire structure of the domain has been set up (largely imported from the foundational ontology), the particular qualities and their types necessary have been specified, paths linking particular entities have been established, and the relations between the physical endurants of the domain and their qualities given. Loading the entire specification into Hets gives a complete structured theory of the domain that has been statically typed checked. Remaining proof obligations that are entailed by the specification are also indicated and addressed at this point. The extract from a Structure Graph shown in Figure 3 was drawn from this complete specification.

Using the account for reasoning about the domain then proceeds by establishing theorems that are to be proved or disproved with respect to the specification. In order to answer our initial question in this example, therefore, we provide a theorem that describes the information need of the user. At present we rely on the linking of CASL and a collection of automatic and semi-automatic theorem provers provided by Hets. In general, and for future applications, we will explore using more specialized reasoners for particular problem areas, such as graph navigation, spatial reasoners, and so on. Even without this specialization, the generic reasoners are still able both to provide proofs for theorems stated within our specification and to deliver particular solutions. These can, in general, require theories from different domains to be ‘combined’.

For the current example, we would want to prove the theorem that there is a path from the Central Station, which is the starting point, and the building the user should try to get to; i.e., we need to prove:

\[ \exists x : \text{Building} \]

• \( PATH_{PED}(Central\_Station, x) \)

where \( x \) is also a building hosting a bank of the required type. When passed to an automatic theorem prover, the structure of our specification then naturally leads it to bring distinct domains together in the search for a proof. The definition of \( PATH_{PED} \) given above states that this predicate holds when there is a \( PATH_{SL} \) holding between the spatial locations that inhere in the physical endurants involved. The definition of \( PATH_{SL} \) then holds when the spatial locations related have positions (in some quality space) such that the positions satisfy the \( path \) predicate. In general, finding solutions here will depend on the mathematical structure of the quality spaces involved. We can, however, prove statements about paths using the general result of the previous section that a view holds between
paths in a route graph and the pre-order relation. While the formal specification in terms of linked lists may not deliver the most performance efficient result, it is certainly sufficient for establishing the correctness or otherwise of the specification and does, in fact, in this case produce several possible solutions.

Finding the particular solution to present to a user would then involve several further issues from way-finding and route instruction—all of which can be successively added to the account given here. Precisely which information is selected is dependent on the physical qualities that are available for forming landmark descriptions, the length of the paths to be admitted, the possibility of discriminating between similar entities in the description, and so on. The dialogue systems with which we are working currently, for example, frequently require dynamically composed theorems of these kinds to be resolved. We can ascertain, for example, that the simpler route description “get out at Tung Station and go to the red building that is not too far away” would not be sufficient because the following implied statements are proved when given to a theorem prover (concretely in this case, SPASS):

\[\text{spec SOME_DOMAIN_THEOREMS =}
\text{BUILDINGS_GRAPHS_AND_COLORS_OF_DOMAIN}
\text{then}
\text{preds query1(x : Building; c1 : CityArea[Building]) ⇔}
(\exists cl : Color • inh(cl, x) ∧ pos(red, cl))
∧ \exists cr : CityArea[Building]
  • (\exists sl : SL • inh(sl, x) ∧ pos(cr, sl))
∧ distance(cl, cr) = not_so_far
\] % (query1 B1) % implied
\[\text{• = query1(B2, TungStation_creg)} % (not query1 B2) % implied
\text{• query1(B3, TungStation_creg)} % (query1 B3) % implied
\]

The predicate \text{query1} is used as shorthand to pick out some building that is red and ‘not too far’ from some city region. The proof status of the subsequent three theorems shows that this holds with respect to the Tung Station city region for both buildings B1 and B3 and not for building B2. Further discriminating information therefore needs to be found—for example, that a given path must be followed (e.g., reaching the square first) or that the building is in a particular city area (e.g., Chinatown).

Finally, the following theorem has a unique solution, the bank that we are trying to describe a path to, and so our path description is complete:

\[\text{spec SOME_DOMAIN_THEOREMS (cont’d) =}
\text{BUILDINGS_GRAPHS_AND_COLORS_OF_DOMAIN}
\text{then}
\text{preds query1 • • [see above]}
\text{query2(x : Building; c1 : CityArea[Building]) ⇔}
\]
6 Conclusions and Outlook

The mechanisms we have illustrated here provide a well founded means of grounding issues arising in spatial reasoning and communication within an ontologically sophisticated specification system. We have shown how it is possible to build up a specification in which distinct perspectives on space can be characterized and related to other kinds of entities necessary for interacting in the spatial domain. We have illustrated several distinct mechanisms for relating distinct ontological modules, ranging from entity-to-entity renamings to general theories that allow back-and-forth mappings across theories. These are all combined in a structurally clean specification framework.

There is, however, clearly very much more to do. We are working on both the formal underpinnings of specifying and discovering theory morphisms and the use of the kinds of specifications we have seen in actual spatial dialogue systems. Many of the particular specifications and theorems we have used as illustrations here will in the end need to be produced automatically, either by directly importing ontological specifications and theory-mapping results or by providing semantic interpretations of natural language dialogue acts. These extensions will also find a natural home, however, in the broad ontological axiomatic framework we have described.

The use of stronger modularization strategies for ontologies in order to bring re-use issues under control is also currently gaining considerable importance in other contexts, such as that of the Semantic Web. Cuenca Grau et al. (2006) describe, for example, some of the benefits and problems of decomposing an ontology into submodules related by connective mappings. Their suggestion that optimizations can result from the fact that the different modules may even adopt different logics is very similar to some of the motivations that underlie our own work. Moreover,
whereas their proposed framework involves different members of the description logic family, in our approach we are not restricted in this way and can consider relations ranging more broadly across any kind of logic supported by CASL and the development environment. In general, ontology modules may be combined in a variety of ways, ranging from the simplest and most widespread relation of ontology inclusion to more complex inter-ontology relationships.

We can also usefully relate what we have described in this paper to many threads currently being followed in the semantic and ontology mapping communities in general. In Stuckenschmidt and Uschold (2005), for example, several positions are set out and problems identified. The explicit answers given in our framework to questions such as the nature of semantic mappings, their status as first class entities, and their organisation and role in an overall account provide concrete proposals for the issues raised. We have linked particular formalizations of space to ways of talking about space and have shown how those formalizations may be placed in relation to one another in terms of declaratively specified theory morphisms. This allows information to be followed through different combinations and to be accumulated in order to reason about spatial problems. We also see the combined theories that are produced by views as standing as one possible instantiation of the formal theory of blends proposed by Goguen (1999). Here we have suggested, for example, that a spatial perspective in terms of nodes in a graph can be related, or blended, with a view in terms of areas. In our current work, we are exploring the nature of these combinations in more detail as well as considering how the selection of particular blends over others is indicated by task and dialogue information.

In summary, we have shown how the kinds of specifications developed here make direct use of and extend well researched and developed foundational ontologies. We see the flexibility and logical foundation of our approach and its combination with practical application brought about by exploring particular formalizations of space for supporting intelligent spatial behavior as an extremely promising way of moving beyond purely theoretical discussion of several critical issues.

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References


Farrar, S. and Bateman, J. (2004). General ontology baseline. SFB/TR8 internal report II-[OntoSpace]: D1, Collaborative Research Center for Spatial Cognition, University of Bremen, Germany.


