Lightweight Temporal Description Logics with Rigid Roles and Restricted TBoxes

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Motivation

Vision: Express temporal knowledge in a DL ontology

Applications: KR & reasoning ... ... over temporal conceptual data models ... in the medical domain

Example: ‘A patient who has diabetes now may develop certain disorders in the future’

Approach: Extend DLs with point-based temporal operators
\[\leadsto\] Temporal description logics (TDLs) [Schild ’93]
The TDL landscape

Previous work

$\mathcal{ALC/EL/DL-Lite} + \mathcal{LTL/CTL} \leadsto \mathcal{PTIME} \ldots \text{undecidable}$


Challenges

- Allow rigid roles to capture time-invariant relations
  
  e.g.: hasBloodGroup, hasGeneticDisease, ...

- With rigid roles, even $\mathcal{EL} + \Diamond$ and $\mathcal{EL} + E\Diamond$ are undecidable!

Goal

Indetify decidable (and tractable) fragments of $\mathcal{EL} + \mathcal{CTL}$
TDLs in a nutshell: syntax

**TDLs are** modal description logics – here $\mathcal{EL}$ + CTL:

\[
C := \top \mid A \mid C \sqcap C \mid \exists r.C \mid E \Diamond C \mid E \Box C \mid \ldots
\]

$\mathcal{EL}$

$\Box$ CTL operators

$\mathcal{ALC}$ + CTL additionally allows $\neg$ (and thus $\sqcup$, $\forall$)

**Example:** $\exists \text{hasDisease}.\text{Diabetes} \sqsubseteq E \Diamond \exists \text{hasDisease}.\text{Glaucoma}$

**Design choices**

- Temporal operators from CTL
- Temporal concepts
- Acyclic TBoxes *(NEW)*
TDLs in a nutshell: semantics

Temporal dimension: worlds + tree-shaped ‘future’ relation

DL dimension: one full DL interpretation per world

- Constant domain assumption
- Rigid roles allowed
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![Diagram](image-url)
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$$\begin{align*}
&\in \quad (A \sqcap E \diamond \exists r. B)^I \\
&(A \sqcap \exists r. E \diamond B)^I
\end{align*}$$
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\equiv \\
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\end{align*}
\]
Contribution

We study decidability and complexity of subsumption

<table>
<thead>
<tr>
<th>Our results</th>
<th>[GJS KR’14]</th>
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</thead>
<tbody>
<tr>
<td>$\mathcal{EL} + \ldots$ empty TBox</td>
<td>acyclic TBoxes</td>
</tr>
<tr>
<td>$\ldots E\bigcirc$ in $\text{PTime}$</td>
<td>in $\text{PTime}$</td>
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<tr>
<td>$\ldots E\lozenge$ in $\text{PTime}$</td>
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<tr>
<td>$\ldots E\bigcirc, E\lozenge$ $\text{coNP-complete}$</td>
<td>in $\text{coNExpTime}$</td>
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<tr>
<td>$\ldots E\lozenge, A\Box$ in $\text{PSpace}$</td>
<td>$\text{PSpace-complete}$</td>
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<tr>
<td>$\mathcal{ALC} + \text{CTL}$</td>
<td>decidable but nonelementary</td>
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- First fragments of $\mathcal{EL}$-based TDLs with rigid roles with elementary (even polynomial) complexity
The 2 main results (out of 4)

**Theorem**

1. $\mathcal{EL} + E\Diamond$ and $\mathcal{EL} + E\bigcirc$ over acyclic TBoxes are in PTIME.
2. $\mathcal{EL} + \{E\Diamond, \ A\Box\}$ over acyclic TBoxes is PSPACE-complete.

**Proof sketch**

1. Build abstract representation of canonical model of input TBox, using 3-phase algorithm (thanks to acyclicity).

2. Upper bound:
   - abstract representation blows up $\leadsto$ consider single traces
   - complete the traces one at a time (think tableaux)
   - polynomial size bound thanks to acyclicity

Lower bound: reduction from QBF
Conclusion

- Acyclic TBoxes can help design well-behaved $\mathcal{EL}$-based TDLs
- $\mathcal{EL} +$ CTL fragments of elementary (polynomial) complexity
- Byproduct: complexity results for positive fragments of product modal logics $K \times K$, $S4 \times K$

- More expressive fragments
e.g., $\mathcal{EL} + \{\text{E} \bigcirc, \text{E} \Diamond\}$ (non-convex) over acyclic TBoxes
- Cyclic TBoxes
- Change the temporal component: LTL, $\mu$-calculus?
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Thank you.