Description Logics:
an Introductory Course on a Nice Family of Logics

Day 5: Justifications

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So far, we have talked a lot about standard reasoning problems

- consistency
- satisfiability
- entailments
- ...is this all that is relevant?

Next, we will look at 1 reasoning problem that

- cannot be polynomially reduced to any of the above standard reasoning problems
- is relevant when working with a non-trivial ontology
- ...justifications!
Imagine you are building, possibly with your colleagues, an ontology $\mathcal{O}$: non-trivial, with say 500 axioms, or 5,000 (NCI has $\geq 300,000$)

(S1) $\mathcal{O} \models C \subseteq \bot$ and you want to know why

(S2) 27 classes $C_i$ are unsatisfiable w.r.t. $\mathcal{O}$
  – imagine $\mathcal{O}$ is coherent, but $\mathcal{O} \cup \{\alpha\}$ contains 27 unsatisfiable classes
  – ...even for a very sensible, small, harmless axiom $\alpha$

(S3) $\mathcal{O}$ is inconsistent
  – imagine $\mathcal{O}$ is consistent, but $\mathcal{O} \cup \{\alpha\}$ is inconsistent
  – ...even for a very sensible, small, harmless axiom $\alpha$

? what do you do?

? how do you go about repairing $\mathcal{O}$?

? which tool support would help you to repair $\mathcal{O}$?
Imagine you are building, possibly with your colleagues, an ontology \( \mathcal{O} \): non-trivial, with say 500 axioms, or 5,000 (NCI has \( \geq 300,000 \))

\[ (S4) \, \mathcal{O} \models \alpha, \text{ and you want to know why} \]

- e.g., so that you can trust \( \mathcal{O} \) and \( \alpha \)
- e.g., so that you understand how \( \mathcal{O} \) models its domain

\[ ? \text{ what do you do?} \]
\[ ? \text{ how do you go about understanding this entailment?} \]
\[ ? \text{ which tool support would help you to understand this entailment?} \]
\[ ? \text{ would this tool support be the same/similar to the one to support repair?} \]
In all scenarios \( (S_i) \), we clearly want to know at least the **reasons for** \( \mathcal{O} \models \alpha \), which axioms can I/should I

(S1) **change** so that \( C' \) becomes satisfiable w.r.t. \( \mathcal{O}' \)?
(S2) **change** so that \( \mathcal{O}' \) becomes coherent?
(S3) **change** so that \( \mathcal{O}' \) becomes consistent?
(S4) **look at** to understand \( \mathcal{O} \models \alpha \)?
Consider the following ontology $\mathcal{O}$ with $\mathcal{O} \models C \sqsubseteq \bot$:

$$\mathcal{O} := \{ C \sqsubseteq D \cap E \quad (1) \\
D \sqsubseteq A \cap \exists r. B_1 \quad (2) \\
E \sqsubseteq A \cap \forall r. B_2 \quad (3) \\
B_1 \sqsubseteq \neg B_2 \quad (4) \\
D \sqsubseteq \neg E \quad (5) \\
G \sqsubseteq B \cap \exists s. C \} \quad (6)$$

Find a justification for $C \sqsubseteq \bot$ in $\mathcal{O}$.
How many justifications are there?
More about Justifications

Facts:
1. for each entailment of $\mathcal{O}$, there exists at least one justification
2. one entailment can have several justifications in $\mathcal{O}$
3. justifications can overlap
4. let $\mathcal{O}'$ be obtained as follows from $\mathcal{O}$ with $\mathcal{O} \models \alpha$:
   - for each justification $\mathcal{J}_i$ of the $n$ justifications for $\alpha$ in $\mathcal{O}$, pick some $\beta_i \in \mathcal{J}_i$
   - set $\mathcal{O}' := \mathcal{O} \setminus \{\beta_1, \ldots, \beta_n\}$
   then $\mathcal{O}' \not\models \alpha$, i.e., $\mathcal{O}'$ is a repair of $\mathcal{O}$.
5. if $\mathcal{J}$ is a justification for $\alpha$ and $\mathcal{O}' \supseteq \mathcal{J}$, then $\mathcal{O}' \models \alpha$.
   Hence any repair of $\alpha$ must touch all justifications.
6. if $\mathcal{O} \models \alpha$, $\mathcal{O} \models \beta$, and
   $\forall$ justification $\mathcal{J}$ for $\alpha \exists$ a justification $\mathcal{J}'$ for $\beta$ with $\mathcal{J}' \subseteq \mathcal{J}$,
   then repairing $\beta$ repairs $\alpha$. 
Let $\mathcal{O} = \{\beta_1, \ldots, \beta_m\}$ be an ontology with $\mathcal{O} \models \alpha$.

Get1Just($\mathcal{O}$, $\alpha$)
Set $\mathcal{J} := \mathcal{O}$ and $\text{Out} := \emptyset$
For each $\beta \in \mathcal{O}$
    If $\mathcal{J} \setminus \{\beta\} \models \alpha$ then
        Set $\mathcal{J} := \mathcal{J} \setminus \{\beta\}$ and $\text{Out} := \text{Out} \cup \{\beta\}$
Return $\mathcal{J}$

Claim: 
- loop invariants: $\mathcal{J} \models \alpha$ and $\mathcal{O} = \mathcal{J} \cup \text{Out}$
- Get1Just(,) returns 1 justification for $\alpha$ in $\mathcal{O}$
- it requires $m$ entailment tests

Other approaches to computing justifications exists, more performant, glass-box (inside reasoner) and black-box (outside).
(S4) 1 justification suffices, but which? A good, easy one...how to find?
(S1-S3) require the computation of all justifications, possibly for several entailments
- even for one entailment, search space is exponential
[(S2)] requires even more:
- who wants to look at $x \times 27$ justifications? Where to start?

$\Rightarrow$ A justification $\mathcal{J}$ (for $\alpha$) is root if there is no justification $\mathcal{J}'$ with $\mathcal{J}' \subset \mathcal{J}$
- start with root justifications, remove/change axioms in them and
- reclassify: you might have repaired several unsatisfiabilitys at once!

- Check example on slide 6: both justifications for $C \sqsubseteq \bot$ are root, contained in 2 non-root justifications for $G \sqsubseteq \bot$
- repairing $C \sqsubseteq \bot$ repairs $G \sqsubseteq \bot$
B0s: NCBO BioPortal, a repository of 250 ontologies, very varied, not cherry-picked

- recent, optimised implementation of $\text{GetAllJust}(\mathcal{O}, \alpha)$
  - behave well in practise
  - can compute one justification for all atomic entailments of B0s
  - can compute (almost) all justifications for (almost) all atomic entailments of B0s

- recent surveys show that B0s have entailments
  - with large justifications, e.g., with 37 axioms and
  - with numerous justifications, e.g., one entailment had 837 justifications
  - for which justifications can often be understood well by domain experts
  - ...for more, see Horridge’s dissertation
Beyond Justifications

• some justification contain superfluous parts
  – that distract the user
  – see example on slide 6
  – identifying these can help user to focus on the relevant parts
  – this has led to investigation of laconic and precise justifications

• there are still some hard justifications that need further explanation
  – e.g., consider \( O = \{ \)
    
    \[
    \begin{align*}
    P & \supseteq \neg M \\
    RR & \supseteq CM \\
    CM & \supseteq M \\
    RR & \equiv \exists h. T S \sqcup \forall v. H \\
    \exists v. \top & \supseteq M \\
    \end{align*}
    \]

  with \( O \models P \supseteq \bot \)
  – this has led to investigation of lemmatised justifications (see next slide)
  with work in cognitive complexity of justifications
Compute $J' = \{\alpha_1, \alpha_2, \alpha_3\}$ so that

$\text{Complexity}(J, \eta) > \text{Complexity}(J', \eta)$
Cognitive Complexity of Justifications: snapshot of a survey

Syntax: ☐ Manchester Syntax ☑DL Syntax

Set

C1 ⊆ C3
  C3 ⊆ C4
  C1 ⊆ a prop1.C5
    prop1 ∈ R
  C5 ⊆ a prop1.C6
  C4 ∩ (¬ prop1.C6) ⊆ C2

Does the above set of axioms entail the following axiom?

C1 ⊆ C2

☐ Yes
☐ Yes, but not sure
☐ Not sure
☐ No, but not sure
☐ No

1See http://tinyurl.com/owlsurvey2012
**Lemmatised Justifications: an example**

**bold:** axioms in $\mathcal{J}$; **normal:** axioms entailed by $\mathcal{J}$; example from [Horridge Dissertation]

Entailment: $\text{Person} \subseteq \bot$

**Person** $\subseteq \neg \text{Movie}$

$\top \subseteq \text{Movie}$

$\forall \text{hasViolenceLevel.} \bot \subseteq \text{Movie}$

$\forall \text{hasViolenceLevel.} \bot \subseteq \text{RRated}$

$\text{RRated} \equiv (\exists \text{hasScript.ThrillerScript}) \sqcup (\forall \text{hasViolenceLevel.High})$

$\text{RRated} \subseteq \text{Movie}$

$\text{RRated} \subseteq \text{CatMovie}$

$\text{CatMovie} \subseteq \text{Movie}$

$\exists \text{hasViolenceLevel.} \top \subseteq \text{Movie}$

$\text{Domain} (\text{hasViolenceLevel, Movie})$
Thanks to

Franz Baader, Samantha Bail,
Sebastian Brandt, Bernardo Cuenca-Grau,
Yevgeny Kazakov, Boris Konev,
Carsten Lutz, Matthew Horridge,
Ian Horrocks, Bijan Parsia,
Boontawee Suntisrivaraporn, Stephan Tobies
Dirk Walther, Frank Wolter

for results and material used here
Some further pointers for reading

- http://dl.kr.org/ for DL proceedings and the DL mailing list
- KR proceedings
- The Description Logic Handbook, Cambridge University Press
- http://www.w3.org/2007/OWL/ for stuff on OWL
  - http://www.w3.org/community/owled/ new community group
- http://owl.cs.manchester.ac.uk/ for stuff on OWL from Manchester
  - http://owl.cs.manchester.ac.uk/about/orientation/a-logics-perspective/
  - http://owl.cs.manchester.ac.uk/tools/
Thanks for your attention!

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