Logics and categories for software engineering and artificial intelligence

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Exercise Sheet 5
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Exercise 5.1 (Logelei)

Consider the following Logelei:

> When I recently took a train, in Bremen, three teenagers came in, a boy and two girls. The boy asked his companions (named Olga and Petra) who from their class who come to the party that was planned. It was an interesting subject for them, although they made only indirect statements.
>
> Olga started: „If neither Bernd nor Christian come, then Nobert won't come either".
>
> Then Petra: „If Dieter and Norbert come, then so will Elgar".
>
> Olga: „If neither Axel nor Lars come, so wont' Christian".
>
> Petra: „If Fabian does not come to the party, then Jürgen won't, provided Martin comes".
>
> Olga: „If both Haug and Axel come, then Christian won't appear".
>
> Petra: „If Martin joints the party, then Bernd will (if Lars does not come) join the party as well".
>
> The trains reached Osnabrück, where the three teenagers left. The boy, who left last, just turned to me and whispered: „Our conversation will have confused you a bit. You have to notice that one of the girls always tells the truth, while the other one never utters a true sentence".
>
> Before I could ask him who the liar was, the boy has left the train. Since then, I speculate who would come to the party and who wouldn't. Who??!

Specify this problem, using alternatively ordinary, free, or cofree specifications. Note that free and cofree specifications require a different style of axioms. Discuss the differences.

Exercise 5.2 (Conservative extension)

Consider the following description logic theories.

(a)

$$
\begin{array}{rcl}
\mathsf{Lecture} & \sqsubseteq & \exists\mathsf{has\_subject.Subject} \sqcap \exists\mathsf{given\_by.Lecturer} \\
\mathsf{Intro\_AI} & \sqsubseteq & \mathsf{Lecture}
\end{array}
$$

Is the following a conservative extension?

$$
\begin{array}{rcl}
\mathsf{Intro\_AI} & \sqsubseteq & \exists\mathsf{has\_subject.Logic} \\
\mathsf{Intro\_AI} & \sqsubseteq & \exists\mathsf{has\_subject.NeuralNetworks} \\
\mathsf{Logic} \sqcap \mathsf{NeuralNetworks} & \sqsubseteq & \bot
\end{array}
$$

(b)

$$
\begin{array}{rcl}
\mathsf{Penguin} & \sqsubseteq & \mathsf{Bird} \\
\mathsf{Bird} & \sqsubseteq & \mathsf{LivingBeing}
\end{array}
$$

Is the following a conservative extension?

$$
\begin{array}{rcl}
\mathsf{Bird} & \sqsubseteq & \mathsf{Animal} \\
\mathsf{Animal} & \sqsubseteq & \mathsf{LivingBeing}
\end{array}
$$

(c)

$$\text{Penguin} \quad \sqsubseteq \quad \text{Bird}$$

Is the following a conservative extension?

$$
\begin{aligned}
\text{Bird} &\quad \sqsubseteq \quad \text{CanFly} \\
\text{Penuin} &\quad \sqsubseteq \quad \neg\text{CanFly}
\end{aligned}
$$

In each case, consider different possible senses of "conservative extension". If applicable, construct a witness concept.