Logik für Informatiker
Logic for computer scientists

Till Mossakowski

WiSe 2007/08

Rooms

- Monday 13:00 - 15:00 GW2 B1410
- Thursday 13:00 - 15:00 GW2 B1410
- Exercises (bring your Laptops with you!)
  - either Monday 13:00 - 15:00 GW2 B1410 and MZH 5210 (Sergey)
  - or Wednesday 8:00 - 10:00 MZH 7250

  . . . this will be decided on Monday 29th October, 13:00 - 15:00 GW2 B1410
- Web:

  `www.informatik.uni-bremen.de/agbkb/lehre/ws07-08/Logik/`

The formal language PL1

PL1 is the formal language of first- order predicate logic

Why do we need a formal language?

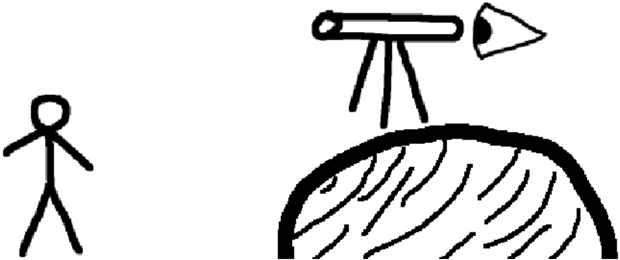
⇒ Slides from Prof. Barbara König, Universität Duisburg-Essen

http://www.ti.inf.uni-due.de/teaching/ws0607/logik/folien/einfuehrung.pdf

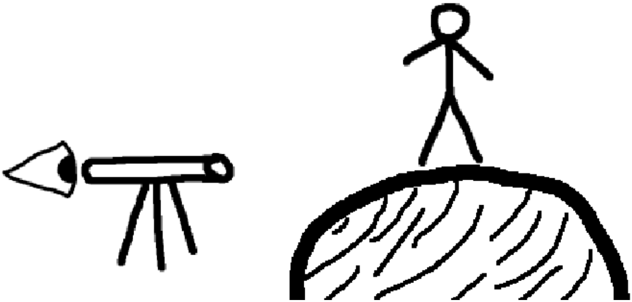
Problem: Natürliche Sprache ist mehrdeutig.

Beispiel:

Ich sah den Mann auf dem Berg mit dem Fernrohr.

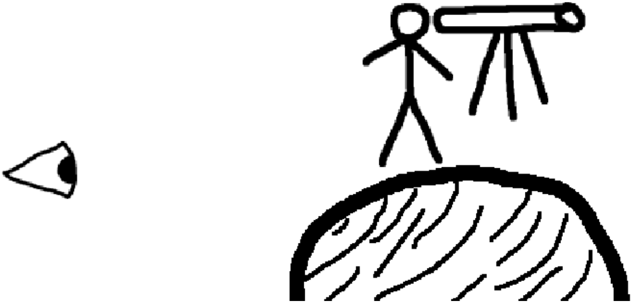


(((Ich sah den Mann) auf dem Berg) mit dem Fernrohr)



((Ich sah (den Mann auf dem Berg)) mit dem Fernrohr)

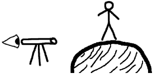
((Ich sah den Mann) (auf dem Berg mit dem Fernrohr))



(Ich sah ((den Mann auf dem Berg) mit dem Fernrohr))

(Ich sah (den Mann (auf dem Berg mit dem Fernrohr)))

(((Ich sah den Mann) auf dem Berg) mit dem Fernrohr)



((Ich sah (den Mann auf dem Berg)) mit dem Fernrohr)

((Ich sah den Mann) (auf dem Berg mit dem Fernrohr))

(Ich sah ((den Mann auf dem Berg) mit dem Fernrohr))

(Ich sah (den Mann (auf dem Berg mit dem Fernrohr)))

5 mögliche Interpretationen!

The language of PL1: individual constants

- Individual constants are symbols that denote a person, thing, object
- Examples:
  - Numbers: 0, 1, 2, 3, . . .
  - Names: Max, Claire
  - Formal constants: a, b, c, d, e, f, n1, n2
- Each individual constant must denote an existing object
- No individual constant can denote more than one object
- An object can have 0, 1, 2, 3 . . . names

The language of PL1: predicate symbols

- Predicate symbols denote a property of objects, or a relation between objects
- Each predicate symbol has an arity that tell us how many objects are related
- Examples:
  - Arity 0: Gate0_is_low, A, B, . . .
  - Arity 1: Cube, Tet, Dodec, Small, Medium, Large
  - Arity 2: Smaller, Larger, LeftOf, BackOf, SameSize, Adjoins . . .
  - Arity 3: Between

The interpretation of predicate symbols

- In Tarski's world, predicate symbols have a fixed interpretation, that not always completely coindices with the natural language interpretation
- In other PL1 languages, the interpretation of predicate symbols may vary. For example, $\leq$ may be an ordering of numbers, strings, trees etc.
- Usually, the binary symbol $=$ has a fixed interpretation: equality

Atomic Sentence	Interpretation
Tet(a)	a is a tetrahedron
Cube(a)	a is a cube
Dodec(a)	a is a dodecahedron
Small(a)	a is small
Medium(a)	a is medium
Large(a)	a is large
SameSize(a, b)	a is the same size as b
SameShape(a, b)	a is the same shape as b
Larger(a, b)	a is larger than b
Smaller(a, b)	a is smaller than b
SameCol(a, b)	a is in the same column as b
SameRow(a, b)	a is in the same row as b
Adjoins(a, b)	a and b are located on adjacent (but not diagonally) squares
LeftOf(a, b)	a is located nearer to the left edge of the grid than b
RightOf(a, b)	a is located nearer to the right edge of the grid than b
FrontOf(a, b)	a is located nearer to the front of the grid than b
BackOf(a, b)	a is located nearer to the back of the grid than b
Between(a, b, c)	a, b and c are in the same row, column, or diagonal, and a is between b and c

Atomic sentences

- application of predicate symbols to constants: Larger(a,b)
- the order of arguments matters: Larger(a,b) vs. Larger(b,a)
- Atomic sentences denote truth values (true, false)

Function symbols

- Function symbols lead to more complex terms that denote objects. Examples:
  - father, mother
  - +, -, *, /
- This leads to new terms denoting objects:
  - father(max)   mother(father(max))
  - 3*(4+2)
- This also leads to new atomic sentences:
  - Larger(father(max),max)
  - 2<3*(4+2)

Logical validity; satisfiability

A sentence A is a logically valid, if it is true in all circumstances.

A sentence A is a satisfiable, if it is true in at least one circumstance.

A circumstance is

- in propositional logic: a valuation of the atomic formulas in the set { true, false }
- in Tarski's world: a block world

Logical consequence

A sentence B is a logical consequence of A, if all circumstances that make A true also make B true.

In symbols: $A \models B$.