## Trace Logic

Trace logic according to Steve Schneider, Concurrent and Real-time Systems, The CSP Approach, John Wiley \& Sons, Ltd, 2000.

## 1 General

| $\Sigma$ | universal set of events |
| :--- | :--- |
| $\checkmark$ | termination event, not in $\Sigma$ |
| $\tau$ | internal event, not in $\Sigma$ |
| $\Sigma^{\checkmark}$ | $\Sigma \cup\{\checkmark\}$ |
| $\Sigma^{\{\checkmark, \tau\}}$ | $\Sigma \cup\{\checkmark, \tau\}$ |
| $A$ | set of eventsA $\subseteq \Sigma^{\checkmark}$ |
| $A^{\checkmark}$ | $A \cup \checkmark$ |
| $a$ | external event from $\Sigma^{\checkmark}$ |
| $\mu$ | external or internal event from $\Sigma^{\{\checkmark, \tau\}}$ |
| $c . v$ | communication event with event v and channel c |
| channel $(c . v)$ | channel c of c.v |
| value(c.v) | value v of c.v |
| seq | sequence |
| $t r$ | finite trace |
| $u$ | infinite trace |
| $s$ | timed trace |
| $T R A C E$ | set of finite traces |
| $I T R A C E$ | set of infinite traces |
| $T T$ | set of timed traces |
| $t$ | $t \in \mathbb{R}^{+}$ |
| $I$ | interval $\in \mathbb{R}^{+}$ |
| $P(a)$ | predicate $P$ |
| $P R O C$ sat $P(t r)$ | $\forall$ tr $\in$ traces (PROC) $\bullet P(t r)$ |

## 2 Untimed Traces and Sequences

| <> |  | empty sequence |
| :---: | :---: | :---: |
| $<a_{1}, a_{2}, \ldots, a_{n}>$ |  | sequence of listed elements |
| $<a \mid a \leftarrow \operatorname{seq}, P(a)>$ |  | sequence comprehension |
| $s e q_{1} \frown \operatorname{seq}_{2}$ |  | sequence concatenation |
| head (seq) |  | first element of seq |
| tail(seq) |  | seq without its first element |
| foot(seq) |  | last element of seq |
| init(seq) |  | seq without its last element |
| \#seq |  | length of seq |
| seq@i |  | ith element of seq |
|  |  | (counting from 0) |
| $\sigma(\mathrm{seq})$ |  | set of events appearing in seq |
| $a$ in seq |  | event a appears in seq |
| term(seq) |  | seq includes $\checkmark$ |
| $s e q_{1}=s e q_{2}$ |  | $\mathrm{seq}_{1}$ and $\mathrm{seq}_{2}$ identical |
| $s e q_{1} \leqslant s e q_{2}$ |  | seq $_{1}$ is prefix of seq ${ }_{2}$ |
| $s e q_{1} \preceq s^{\text {seq }}{ }_{2}$ |  | seq $_{1}$ is subsequence of seq $_{2}$ |
|  |  | (not necessarily contiguous) |
| $s e q$ interleaves $s e q_{1}, s e q_{2}$ |  | seq is an interleaving of sequences |
|  |  | $\mathrm{seq}_{1}$ and seq ${ }_{2}$ |
| seq synch $_{\text {A }}$ seq $_{1}, \operatorname{seq}_{2}$ |  | seq synchronizes seq d $_{1}$ and seq ${ }_{2}$ |
|  |  | on events in $\mathrm{A}^{\checkmark}$ |
| $s e q \upharpoonright A$ |  | subsequence of elements of seq |
|  |  | in A |
| $s e q \backslash A$ |  | subsequence of elements of seq |
|  |  | not in A |
| $s e q \downarrow A$ | $\equiv \#(s e q \upharpoonright A)$ | number of elements of A in seq |
| channels ( $t r$ ) | $\equiv\{\operatorname{channel}(x) \mid x$ intr $\}$ | set of channels in trace |
| $t r \Downarrow c$ | $\equiv<\operatorname{value}(x) \mid x \leftarrow$ tr, channel $(x)=c>$ | sequence of values c in trace |
| $\operatorname{tr} \Downarrow C$ | $\equiv<\operatorname{value}(x) \mid x \leftarrow$ tr, $\operatorname{channel}(x) \in C>$ | sequence of values in C in trace |

## 3 Timed Traces

| $s \upharpoonright A$ |  | s restricted to $\mathrm{A}:<(\mathrm{t}, \mathrm{a}) \mid(\mathrm{t}, \mathrm{a}) \leftarrow \mathrm{s}, \mathrm{a} \in \mathrm{A}>$ |
| :---: | :---: | :---: |
| $s \backslash A$ | $\equiv s \upharpoonright \Sigma \backslash A$ | subsequence of elements of seq not in A |
| $s \downarrow A$ | $\equiv \#(s \upharpoonright A)$ | number of elements of A in s |
| strip(s) | $\equiv<a \mid(t, a) \leftarrow s>$ | $s$ with times removed |
| $s+t$ | $\left.<\left(t^{\prime}+t, a\right) \mid\left(t^{\prime}, a\right) \leftarrow s\right)>$ | s delayed by t |
| $s-t$ | $<\left(t^{\prime}-t, a\right) \mid\left(t^{\prime}, a\right) \leftarrow s, t^{\prime} \geqslant t>$ | s brought earlier by t |
| begin(s) |  | time of the first event in s (and $\infty$ for the empty trace) |
| end(s) |  | time of the last event in s (and 0 for the empty trace) |
| first(s) |  | first event to appear in $s$ |
| last(s) |  | last event to appear in s |
| $s \uparrow I$ | $<(t, a) \mid(t, a) \leftarrow s, t \in I>$ | s during I |
| $s \\| t$ | $s \uparrow[0, t)$ | $s$ strictly before t |
| $s \upharpoonright t$ | $s \uparrow[0, t]$ | s before t |
| $s \uparrow \mid t$ | $s \uparrow(t, \infty)$ | s strictly after t |
| $s \upharpoonleft t$ | $s \uparrow[t, \infty)$ | s after t |

## 4 Macros for Timed Traces

$$
\begin{aligned}
a \text { at } t & \equiv<(t, a)>\preceq s & & \text { a occurres at time t } \\
a \text { at } I & \equiv \exists t \in I \bullet a \text { at } t & & \text { a occurs in interval I } \\
A \text { at } I & \equiv \exists a \in A \bullet a \text { at } I & & \text { an Event from A occurres in interval I } \\
\text { no } A & \equiv \neg(A \text { at }[0, \infty)) & & \text { no events from A appear in s } \\
\text { only } A & \equiv \neg\left(\Sigma^{\checkmark} \backslash A \text { at }[0, \infty)\right) & & \text { only events from A appear in s }
\end{aligned}
$$

