State Observability through Prior Knowledge: A Conceptional Paradigm in Inertial Sensing

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Abstract: Inertial Navigation Systems suffer from unbounded errors on the position and orientation estimate. Exteroceptive sensors may not always be available to correct the error. Applications in the literature overcome this problem by fusing IMU data with prior knowledge in an ad-hoc fashion. In different applications, various knowledge is available, which allows to correct the erroneous state estimate. In this position paper, we argue that the fusion of knowledge and inertial sensor data should be viewed as a paradigm and that the observability of systems with prior knowledge should be analysed theoretically. With a theoretical foundation, application design will be simplified and verifiable. We show methods to start the analysis and give a first proof with practical insight.

1 INTRODUCTION

Inertial Navigation Systems (INS) perform dead reckoning on the measurements of Inertial Measurement Units (IMU). This way, the state of an object, consisting of position, orientation and velocity, is estimated. Dead reckoning has the major disadvantage that the estimate's error increases over time. This drift, e.g. a position error of 1.5 m after only 5 s of dead reckoning (Wenk, 2017, Figure 3.7), is caused by accumulating the measurement errors of the IMU. To correct the drift, IMU measurements are often fused with exteroceptive position sensors, for example the GPS.

Given this drift, it seems necessary to have an exteroceptive sensor. However, in fact drift-free estimates can be obtained from IMU measurements alone if appropriate scenario knowledge is exploited. This surprising realisation motivates our position paper.

There already exist several works that successfully demonstrate the usefulness of prior knowledge in state estimation. The pose (position and orientation) of humans walking in buildings can be tracked using prior knowledge instead of exteroceptive sensors (Harle, 2013; Beauregard et al., 2008; Woodman and Harle, 2008). The estimate errors of the pose are bounded, which means that the estimates are drift-free. Intuitively, we define that as observability, and we will further refer to states that can be estimated with bounded errors as observable.

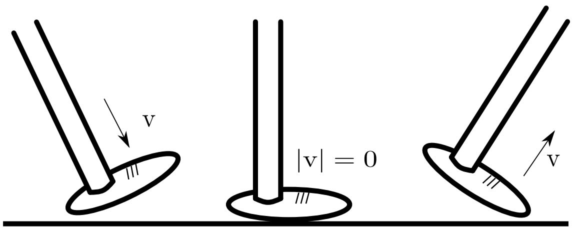

Figure 1: The foot has 0 velocity when it stands on ground.

The indoor tracking works achieve state observability through prior knowledge about human motion. While walking, the feet periodically touch the ground, wherefore they have 0 velocity at one moment (see Fig 1). Updating the velocity with this information is called a Zero-Velocity-Update (ZUPT) and makes the velocity observable (Foxlin, 2005).

The ZUPTs are used to estimate step lengths, which are fed to a Particle Filter (PF) as the dynamic update. The PF models the knowledge that humans can not pass through walls by removing particles that cross walls. The used prior knowledge makes the pose observable and even enables localization without knowing the starting pose (Woodman and Harle, 2008). More surprisingly, the use of the prior knowledge enables to map buildings, using an IMU as the only sensor (Angermann and Robertson, 2012).

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Several other works achieve state observability by combining IMU data with knowledge. Dissanayake et al. (2001) uses the knowledge, that a wheeled vehicle only moves in forward direction. It is shown that velocity and inclination are observable if the vehicle drives a curve. In attitude estimation, several works use the gravity vector to observe roll and pitch (Vaganay et al., 1993; Rehbinder and Hu, 2004; Sabatini, 2006). Wenk (2017, Sec. 3.9) shows that this is equivalent to the prior that the acceleration is 0 in average. The motion suit of Xsens fuses the measurements of 17 IMUs with the knowledge that they are linked by the human body joints (Roetenberg et al., 2009). This enables to observe the angles of all body joints even without a magnetometer (Wenk, 2017, Sec. 4.4). Applications with exteroceptive sensors incorporate knowledge to improve the observability of the state (López-Araquistain et al., 2019; Xu et al., 2016; Battistello et al., 2012).

In most cases, the improvement of the estimate is shown empirically. Instead, the state observability can be analysed from a theoretical point of view. The theoretical analysis has the advantage that it reveals whether the knowledge reduces the state drift or eliminates it. This difference is critical, since a reduced drift still causes an error on long term measurements.

It can be investigated which knowledge makes which state observable. This may give further insight about its use and benefits. More importantly, error cases may be revealed before an application is tested.

The use of prior knowledge is highly beneficial, but only understood on a per-case basis. It is mainly evaluated application specific. Therefore, it misses theoretical foundation for general applications. So we argue that fusing prior knowledge with IMU data should be viewed as a conceptional paradigm and investigated from a theoretical point of view.

The remainder of the paper is structured as follows. In Section 2, we explain the gain from understanding the paradigm of state observability through prior knowledge. We will show the structure of prior knowledge and algorithms to use it. In Section 3, we will show a method to analyse state observability. We give an example of its usage in form of a first proof and show how to design an application in track cycling based on the paradigm. At last, we state the major research goals of our project in Section 4.

2 EXPLAINING THE NEED FOR A PARADIGM

In application design regarding sensor fusion, we want to know whether we can estimate the states we require. The estimate error of unobservable states increases over time, whereas the error of observable states is bounded. Hence, we want to achieve observability of relevant states.

2.1 State Observability

Position: "If we understand the state observability through prior knowledge, we can predict the error behaviour of the estimate."

For many applications, prior knowledge that may lead to observability of the relevant states is available. State observability analysis reveals whether the knowledge is sufficient to observe the state.

An example of observability analysis can be seen in (Dissanayake et al., 2001). They use the forward velocity prior (see Figure 2). It states that the wheeled vehicle only drives forward with no side slip. This can be modelled by:

$$\vec{v}_b = \begin{pmatrix} |\vec{v}_w| & 0 & 0 \end{pmatrix}^T \quad (1)$$

where $\vec{v}_b$ is the velocity in body coordinates and $\vec{v}_w$ the velocity in world coordinates. Whenever the vehicle drives a curve around the y or z axis, the forward velocity is observable. Additionally, the sensor biases are observable (Rothman et al., 2014).

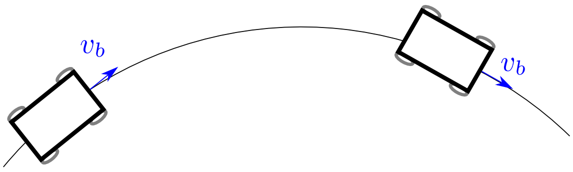

Figure 2: Forward velocity prior on a wheeled vehicle.

The collected insight enables us to predict the error behaviour, instead of measuring it. We can predict that the velocity error is low while driving curves and high otherwise. The observations of (Dissanayake et al., 2001) match this prediction.

As shown, the analysis allows to predict the error behaviour of the state estimate. Hence, we could validate the quality of the state estimate before we test it. Therefore, we need the analysis of state observability through prior knowledge.

Position: "Predicting the error behaviour of a state estimator is a powerful tool, which points out knowledge that yields state observability."

The effect of prior knowledge is dependent on the state configuration. For example, the forward velocity prior yields observability of the velocity for a vehicle driving on a curved line (see Figure 2), but not on a straight line (see Figure 3). Hence, fusing IMU data with prior knowledge may not yield state observability in all state configurations.

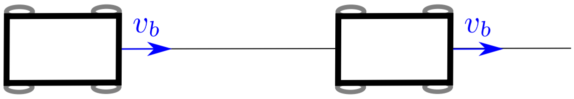

Figure 3: Vehcile driving a straight line. The velocity is not observable with the forward velocity prior.

The observability analysis reveals the observable and unobservable state space configurations. A domain expert can evaluate how likely the unobservable configurations occur. The unobservable configurations may be isolated points in the state space, which are surrounded by observable configurations. Consider a smooth transition between a left and a right curve. The transition point itself is straight, wherefore the velocity is unobservable with the forward velocity prior. However, the velocity can be dead reckoned at that single point. The accumulated error is negligible.

If the vehicle drives on a straight line during the whole application, the velocity is unobservable. In other words, the application takes place in unobservable configurations only. This is the other extreme, which makes the prior knowledge useless.

When we evaluate how likely unobservable configurations occur in the application, we use additional prior knowledge about the application. The evaluation reveals which additional prior renders the unobservable configurations impossible. This prior either has to be modelled explicitly, or already makes the state observable implicitly. In both cases, the observability analysis points out which assumptions are needed for state observability.

Overall, the analysis of the observability shows, whether the application can work. Hence, it can be used for verification of the system design. This is the most important part of the paradigm of state observability through prior knowledge. We have to understand how using knowledge corrects the drift or only improves the state estimate and when it is useless or even disruptive. In the end, the better the knowledge is understood, the greater the benefit is.

2.2 Types of Knowledge

Position: "Prior knowledge is structured in several groups with different observability characteristics."
In the literature, we find various types of prior knowledge. Different knowledge constrains different states of the state vector. Building plans (Beauregard et al., 2008), vessel routes (Battistello et al., 2012), flight corridors (Xu et al., 2016) and road maps (López-Araquistain et al., 2019) constrain the position of the object. ZUPTs (Foxlin, 2005) and the forward velocity prior (Dissanayake et al., 2001) constrain the velocity. The low acceleration prior (Wenk, 2017) constrains the acceleration.

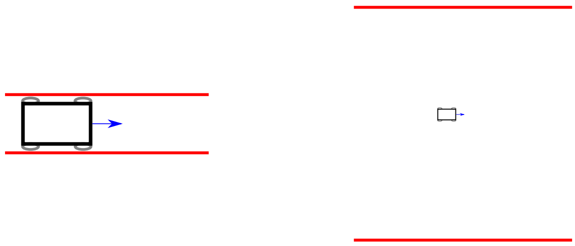

Figure 4: Upper and lower y-position bounds (red). Left is almost an equality constraint. Right has a minor effect.

In many cases, the knowledge can be modelled as equality or inequality constraint. Equality constraints reduce the dimension of the state space and occur if the state is overparametrized. An example is the quaternion parametrization of rotations. It uses four states that are constrained to have norm one, instead of the three rotation angles.

Inequality constraints reduce the state space, but not its dimension. They often occur in the form of upper or lower bounds on a state. Building maps, as they are used in PFs (Harle, 2013), are a representation of complex inequality constraints. They model that the object's position does not equal the position of a wall. Particles that violate the constraint are deleted.

In contrast to equality constraints, inequality constraints can have different strength (see Figure 4). The strength of the constraint affects the observability of the state. Close bounds are almost an equality constraint. However, if the bounds are far apart from each other, the constraint has a minor effect.

In general, constraints are modelled imperfectly. They are either simplified or known inexactly. The imperfection can be grouped into inaccuracy, incompleteness or inconsistency (Podt et al., 2014). An imperfect constraint can be modelled as a probability distribution. However, constraints are often assumed to model the reality perfectly.

The imperfection of a constraint affects the observability of the system. Consider the perfect constraint $\dot{x} = 0$. x stays at its starting value and is observable. A similar imperfect constraint $\dot{x} = \mathcal{N}(0, \sigma^2)$ allows accumulating changes of the state. Hence, it does not yield observability.
Position: "The structure of prior knowledge may be exploited to proof observability for general systems."
Knowledge may be grouped by the type of constraint it imposes on the system. This allows to investigate the state observability in a general fashion. Different equality constraints reduce the dimension of the state space in a similar manner. Hence, the observability can be investigated for general state space descriptions, such as 2D- or 3D-systems.

In the case of INS, the dynamic equations make the observability of the states dependent. If the position is observable, the velocity can be derived by differentiation. There may be other observability relations between the states.

A general view on the types of knowledge enables a fast observability analysis without proofs for applications. With rules and proofs for general systems and types of knowledge, only the structure of the available knowledge has to be determined. The observability characteristics can be derived intuitively by analysing the observability for each knowledge followed by deriving the implied observability. This allows to use concepts based on the paradigm of state observability through prior knowledge without proofing the observability mathematically.

Position: "Structural similarities in prior knowledge may reveal new possible applications."

Many assumptions can be transferred on other applications. For example, the forward velocity prior for vehicles is valid for bikers as well. If observability of a prior knowledge has been proofed for one application, other suitable applications are likely to exist. The observability proof can be generalized for similar applications. Hence, the discovery of a prior that yields observability reveals that similar applications are possible, which were previously thought to be impossible.

2.3 Algorithms for Prior Knowledge

Position: "A better comprehension of the structure of prior knowledge may enable new algorithms."

Using constraints algorithmically in state estimation is already a topic of research and reviewed in (Simon, 2010; Rasool, 2018). Several methods project either the state estimate, the Kalman gain or the whole system on a constrained subspace. On the subspace, the constraint is guaranteed to hold.

A common approach is the use of constraints as pseudo measurements (Tahk and Speyer, 1990). The constraint is handled as a measurement with a constant measurement observation. This allows an easy integration in all variants of Kalman filters. The approach can be used with perfect and imperfect constraints.

In principle, the algorithms search the most probable solution. This can be formulated as a least squares minimization problem. The Moving Horizon Estimator solves the least squares problem for a moving window on the measurements. It can deal with highly non-linear systems and constraints.

The algorithms solve the state estimation problem for different assumptions about the system description and constraints. The performance of each algorithm depends on the (non-)linearity of the system and the constraints, and the type of the constraints. Simon (2010) proposes a decision chart to choose the algorithm based on these parameters. If we understand the structure of prior knowledge in greater detail, we can evaluate the algorithm's performance with regard to other parameters. Hence, we may choose a better algorithm or develop a new one that exploits the peculiarities of the knowledge.

3 OBSERVABILITY ANALYSIS OF PRIOR KNOWLEDGE

In the Introduction, we defined an observable state as a state that can be estimated with bounded errors. The estimate of an observable state, does not drift away from the true value. This definition is relevant for applications, because we generally need estimates of the pose that are always close to the object's real pose.

For theoretical analysis of prior knowledge, we will make use of the more formal observability definitions in (Adamy, 2018) and (Kou et al., 1973) taken from control theory. Both works assume a system definition as follows:

The system has an unknown state x. The derivative of the state depends on the known input u and can be calculated by:

$$\dot{x} = f(x,u) \tag{2}$$

with the known function f. The system has an measurable output z defined by:

$$z = h(x) \tag{3}$$

with the known function h. State, input and output are vectors with arbitrary dimension.

Adamy (2018) defines two forms of observability. The state is strongly observable if it can be derived from the in- and outputs of one time point, i.e. without any data from previous or following time points. In contrast, the state is weakly observable if it can be derived from the in- and outputs of a time interval. We will mainly use weak observability, since the strong one can not be applied on most non-linear systems.

Kou et al. (1973) defines observability similar to uniqueness. The state is observable if there is only one trajectory, which starts at the known start state $x_0 = x(t_0)$ and fits the out- and inputs since the time point t_0. Any ambiguity yields the state unobservable. We will use this definition to further analyse why a state is unobservable and to give advice which prior knowledge would make the state observable.

The strength of observability can not be expressed by the notation of observable or unobservable. Another notation is shown in (Han and Wang, 2008), where a Degree of Observability (DoO) is computed based on the covariance of the state. The continuous DoO takes sensor noise into account. Consider a wheeled vehicle with the forward velocity prior. If the gyrometer error is considerably higher than the rotation axis change of a curve, it dominates the measurement. Hence, a system can be analytically observable, whereas the real application is not. In that case, the DoO would show poor observability for the velocity. It has to be investigated, how the DoO can be predicted from the system parameters.

The weak observability after (Adamy, 2018) can be investigated for prior knowledge that can be modelled as an equality constraint. The main approach is to observe the states by the time derivatives of the system's output equation $z = h(x)$. This results in an observation vector $Z(x)$ of dimension n, where n is the state's dimension.

$$Z(x) = \begin{pmatrix} h(x) \\ \dot{h}(x) \\ \vdots \end{pmatrix} \tag{4}$$

The state is observable if x can be derived from $Z(x)$, which is possible when Z is left invertible:

$$x = Z^{-1}(Z(x)) \tag{5}$$

Z is left invertible at x if its Jacobian is full ranked at x_0 which yields the weak observability criteria:

$$\text{rank}\left(\frac{dZ}{dx}(x_0)\right) = n \tag{6}$$

Evaluating the Jacobian at a state configuration x_0, results in a local form of the observability. If the system has an equal form at multiple configurations (see Figure 5), the correct state can not be determined.

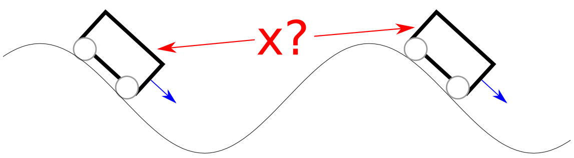

Figure 5: A system with repeating slopes. With the weak observability criteria only, the correct state is unknown.

3.1 "Rollercoaster" Observability Proof

We show the application of the observability analysis in a simple theoretical example. We will proof that a one dimensional system is weakly observable if the rotation axis changes. The proof is a first step towards observability analysis for complex systems.

Consider a rollercoaster. It can only ride on the track, which defines position and orientation. In other words, its pose x is a function of the variable λ:

$$x = f(\lambda) \tag{7}$$

All derivatives of the pose, such as the cartesian and angular velocities, depend on λ and its derivatives:

$$\dot{x} = \dot{\lambda} \cdot f'(\lambda) \tag{8}$$

$$\ddot{x} = (\dot{\lambda})^2 \cdot f''(\lambda) + \ddot{\lambda} \cdot f'(\lambda) \tag{9}$$

$\dot{\lambda}$ is the velocity on the track and $\ddot{\lambda}$ the acceleration.

Theorem 1: A system which pose x only depends on the variable λ is weakly observable from gyrometer measurements if the rotation axis of the orientation is changing.

Proof: We parametrize the orientation q as a quaternion. The orientation quaternion only depends on λ.

$$q = q(\lambda) \tag{10}$$

$$\dot{q}(\lambda) = \dot{\lambda}\frac{dq}{d\lambda} \tag{11}$$

$$\frac{dq}{d\lambda} = \frac{1}{2} q(\lambda) * \omega(\lambda) \tag{12}$$

Where $\omega(\lambda)$ is the angular velocity as a quaternion with 0 real part

$$\dot{q}(\lambda) = \dot{\lambda} \cdot \frac{1}{2} q(\lambda) * \omega(\lambda) \tag{13}$$

With the gyrometer we measure $z = \dot{\lambda}\omega(\lambda)$

$$z = \dot{\lambda}\omega(\lambda) = 2q(\lambda)^{-1} * q(\dot{\lambda}) \tag{14}$$

$$\frac{dZ}{dx}(\lambda) = \begin{pmatrix} \frac{dz}{d\lambda} & \frac{dz}{d\dot{\lambda}} \end{pmatrix} \tag{15}$$

Now we proof the full rank of the matrix via linear independency of the columns:

$$0 \stackrel{!}{=} k_1 \frac{dz}{d\lambda} + k_2 \frac{dz}{d\dot{\lambda}} \tag{16}$$

$$= k_1 \dot{\lambda} \cdot \omega'(\lambda) + k_2 \cdot \omega(\lambda) \tag{17}$$

Equation 17 shows that if $\dot{\lambda}$ is 0, the Jacobian is singular. This is the trivial case, where the object does not move. In this case, no system can be observed from the gyrometer. The other case is when $\omega'(\lambda)$ and $\omega(\lambda)$ are collinear, meaning that the rotation axis does not change its direction. Hence, the angular velocity and acceleration rotating around the same axis is the only non-trivial unobservable case. This only occurs at state configurations where there is either no rotation or the rotation axis is constant. Therefore, the system is observable if the rotation axis changes.

The gathered insight supports the intuitive observability analysis. Observability can be analysed for the rollercoaster in Figure 6 without any calculations. Between the red lines, the rotation axis changes, wherefore the segment is observable. The other segments are unobservable, because the rollercoaster turns around a constant axis only. One would expect to observe a transition from a straight part to a curved part due to the angular velocity change. But the transition would result in a similar angular rate signal as starting at 0 velocity on a curve. Thus, it can not be observed if the velocity can be 0.

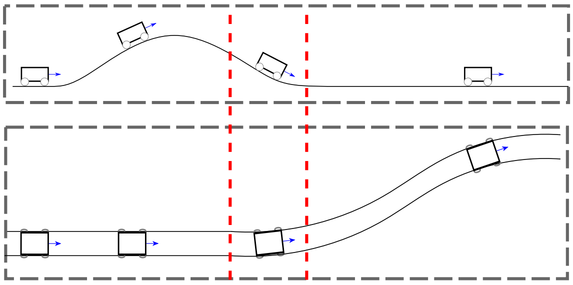

Figure 6: Side view (top) and top-down view (bottom) of a rollercoaster. The dashed red lines mark the segment with weak observability after Theorem 1.

Theorem 1 reveals local observability conditions for general 1D systems. Based on this, we assume that observability analysis can be performed on more complex general systems. For example, a generalisation of Theorem 1 for systems of higher dimension, where the prior knowledge can be modelled as an equality constraint of the form:

$$z = f(x, y) \tag{18}$$

could be applied in sports like track cycling or Formula 1. In these sports, the position can be expressed with two parameters and the orientation with one.

3.2 Track Cycling Design Example

The focus of our research project are theoretical proofs as in Theorem 1, which give insight about observability conditions for prior knowledge. Nevertheless, we want to show how the gathered insight can be used in real world scenarios. The forward velocity prior (Dissanayake et al., 2001) directed us to track cycling (see Figure 7), which has similar conditions as the wheeled vehicle. We will argue that the pose of a track cycler is likely to be observable if available prior knowledge is fused with IMU data, due to the track's shape.

Figure 7: Tracking a biker with an IMU as the only sensor may be possible with prior knowledge.

At track cycling, bikers run a race on a track. It is desired to track their velocity and when they drive in the slipstream of other bikers. To detect whether a biker drives in the slipstream, the positions of all bikers are required. The values should be retrieved by using an IMU at each bike and prior knowledge.

Following the paradigm, we gather knowledge about the dynamics and constraints of the motion. At track cycling, both wheels stay on the symmetrical track. A typical track is shown in Figure 8. The track is taken counter clockwise. The bikes drive forward with almost no side slippage, similar to wheeled vehicles. A biker's speed is limited. Bikers lean into the curve. We measure the local gravity vector with the IMU. The starting position of the bikers is known.

With this still incomplete list of knowledge, we try to analyse the observability of the state. We already know from (Dissanayake et al., 2001) that the velocity is observable when we drive a curve with the forward velocity prior. The bikers follow the track, which contains 2 curves and 2 straight segments. Hence, the velocity is observable periodically.

Since the wheels of the bike have to be on the track, the position is constrained and has only 2 DOF. The height can be calculated from the x and y position. We try to use Theorem 1 to make assumptions about the observability. The theorem states that a 1D system is observable from gyrometer measurements alone if the rotation axis changes. At a single round of track cycling, the biker roughly follows a 1D path on the track's 2D surface. In the curves of this path, the rotation axis changes, which yields observability after Theorem 1.

At the straight track parts, dead reckoning has to be performed. In principle, the velocity and position error will grow unbounded, but only until the biker drives a curve. Hence, the error growth is practically bounded, depending on the time the biker drives on the straight part.

The change of rotation axis can be surely detected,

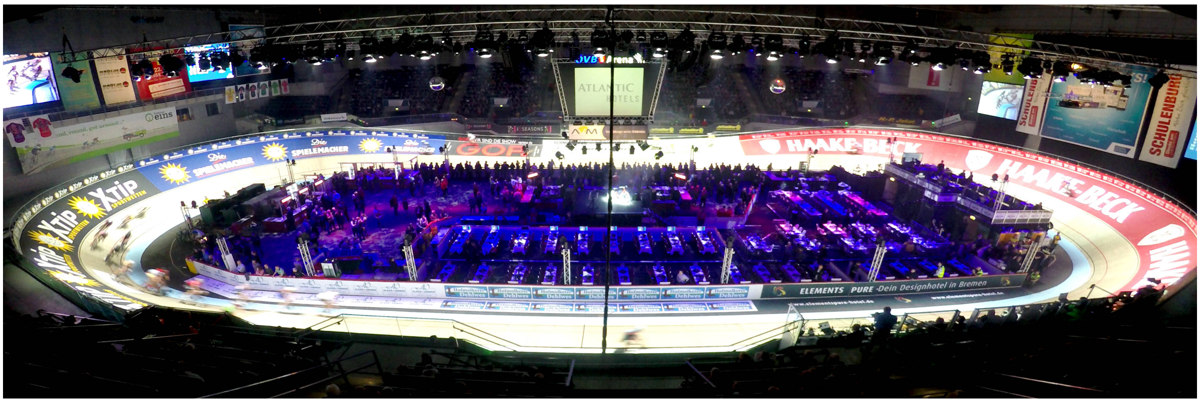

Figure 8: The track of the Sixdays Bremen.

but the biker can be in either curve. Thus, if the starting position is unknown, the biker can not be localized uniquely. This ambiguity can be resolved since the starting position is known. However, the state estimator can not recover if it looses the position once.

At long term, roll and pitch are observable due to the known gravity vector. Since the bikers drive only counter-clockwise, the yaw can be constrained to follow the path of the track.

The analysis of the state observability shows that the relevant states, the position and the velocity, can be expected to have bounded errors if the IMU data is fused with the knowledge. Therefore, we expect that the application is possible, which will be evaluated in a future publication.

The theoretical insight on the prior given by Theorem 1 and (Dissanayake et al., 2001) revealed the behaviour of the estimate error. It has to be investigated whether Theorem 1 can be generalized to the 2D case, as it was used in this example, to back up the approximate argument in this example.

4 CONCLUSION AND FUTURE WORK

The concept of fusing IMU data with prior knowledge is already used in the literature. Surprisingly, various works report bounded errors on normally unobservable states. Their success shows that prior knowledge has the potential to make states observable. However, only a few works provide a theoretical foundation for the observability.

By analysing the observability of states in applications with prior knowledge, we can predict the error behaviour of the state estimate. Thus, applications can be verified before testing. Possible failure cases can be predicted from the observability conditions revealed by the analysis.

We have shown suitable methods, taken from the field of control theory, to analyse the state observability in applications with equality constraints. We started to investigate the observability conditions for simple systems. At a first shot, we found Theorem 1, which states that a 1D system is observable if its rotation axis changes. The method can be used to validate the design of applications before testing it.

Prior knowledge is structured into groups with different observability characteristics. Analysing the groups will result in a better comprehension of prior knowledge. The comprehension can point out applications that were thought to be impossible with IMUs alone, such as the tracking of bikers at track cycling.

The utility of prior knowledge depends on the required estimation accuracy, modelling errors and the sensor noise. This results in different grades of state observability through prior knowledge. Theoretical observability alone, can only guarantee bounded errors for precise models of the real world applications. It has to be investigated how the utility of prior knowledge can be estimated for imperfect conditions.

The paradigm of state observability through prior knowledge aims at understanding the influence of prior knowledge on the observability of the state estimate. With the investigation of the paradigm, we expect to simplify the analysis of the state observability by proofing observability for structural groups of prior knowledge. The gained theoretical insight will result in faster application development and enables verification of state estimation systems.

In our research project, we focus on the theoretical foundation of prior knowledge. We will further analyse general system descriptions, such as 2D systems. We will derive and investigate common structures in prior knowledge from existing applications to proof observability in generalized cases. Our results will be interpreted from an intuitive point of view. This will allow a wide audience to use our results without executing the analysis themselves.

The theoretical results will be accompanied by application examples from the field of sports science. Since most sports follow a rulebook, various kinds of prior knowledge can be applied. Player policies can be used to investigate vague prior knowledge.

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