Universität Bremen

Fachbereich 03 - Informatik

Logic Synthesis

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WORK SHEET 2

Problem 1. Consider the algebra on the set $A = \{0, 1, a\}$ that is defined by the following tables.

$\vee$	0	1	a
0	0	1	a
1	1	1	1
a	a	1	a

$\cdot$	0	1	a
0	0	0	0
1	0	1	a
a	0	1	a

x	$\overline{x}$
0	1
1	0
a	a

Check whether each of the axioms in the Huntington's postulates holds.

Problem 2. Show that the following two algebras are isomorphic to each other.

Algebra 1: $\langle A, \vee, \cdot, \bar{\ }, O_A, I_A \rangle$. Let A be the positive integers that are divisors of 120. For each $x, y \in A$, define the following:

$y \cdot y = GCD(y, x)$: the greatest common divisor of x and y.
$x \vee y = LCM(x, y)$: the least common multiple of x and y.
$\overline{x} = 120/x$: the quotient of 120 divided by x.
$I_A = 120$, $O_A = 1$.

Algebra 2: $\langle B, \vee, \cdot, \bar{\ }, O_B, I_B \rangle$. Let $B = \{0, 1, 2, 3\} \times \{0, 1\} \times \{0, 1\}$. Then, the vector $x = (x_1, x_2, x_3)$ that satisfies $x_1 \in \{0, 1, 2, 3\}$, $x_2 \in \{0, 1, 2\}$, and $x_3 \in \{0, 1, 2\}$ is an element of B. Let $x = (x_1, x_2, x_3)$ and $y = (y_1, y_2, y_3)$ be elements of B, define the the algebra as follows:

$y \cdot y = (\min(x_1, y_1), \min(x_2, y_2), \min(x_3, y_3))$.
$x \vee y = (\max(x_1, y_1), \max(x_2, y_2), \max(x_3, y_3))$.
$\overline{x} = (3 - x_1, 1 - x_2, 2 - x_3)$.
$I_A = (3, 1, 1)$, $O_A = (0, 0, 0)$.

Problem 3. In a Boolean algebra, prove the following without using truth tables: If $a \vee b = a \vee c$ and $ab = ac$, then $b = c$.

Problem 4. Prove that an arbitrary n-variable logic function has an SOP with no more than 2^{n-1} products.

Problem 5. Represent the logic function $f = x(y \oplus z)$ by the canonical SOP, canonical POS, and the positive polarity Reed-Muller expression.

Problem 6. Prove or show a counterexample for the following proposition:

$$x \oplus (y \vee z) = (x \oplus y) \vee (x \oplus z).$$

Problem 7. Calculate the maximum number of nodes in a ROBDD with 10 variables. Show your work!

Problem 8. Calculate the maximum number of nodes in a ROBDD with edge negations for 10 variables. Show your work!