Universität Bremen

Fachbereich 03 - Informatik

Logic Synthesis

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WORK SHEET 3

The followng data structures are used in the first three questions.

```
node{
   int i;          // variable index
   int low;        // low subtree
   int high;       // high subtree
   int count;      // reference counter
   int forward;    // next node on this level
   int backward;   // previous node on this level
   int next_free;  // next node on the free list
}

int perm[n];   // permutation of variable indices
int level[n];  // node number of the first node on each level
int free_list; // frist free node
```

The following procedures are used to manage the list which links the nodes for each level.

```
Add_to_level(u){
   T[u].forward = level[T[u].i];
   if(level[T[u].i] != 0)
      T[level[T[u].i]].backward = u;
   level[T[u].i] = u;
   T[u].backward = 0;
}

Remove_from_level(u){
   if(T[u].forward != 0)
      T[T[u].forward].backward = T[u].backward;
   if(T[u].backward != 0)
      T[T[u].backward].forward = T[u].forward;
   else
      level[T[u].i] = T[u].forward;
}
```

NOTE: Mk must be modified such that when MK allocates a new node, it is added to the level (by a calll to `Add_to_level(u)`). Additionally, every call to MK will automatically call `Ref(u)`.

Problem 1. Write the pseudo-code for the procedure `Ref(u)`, that will increase the reference count for the node u.

```
Ref(u){
   T[u].count++;
}
```

Problem 2. Write the pseudo-code for the procedure `RecursiveDeref(u)`, that will recursively decrease the reference count for the node u and its decedents. When the reference count of a node reaches zero, it must be added to the $freelist$.

```
RecursiveDeref(u){
   T[u].count--;
   if(T[u].count == 0) {
      Add_to_freelist(u);
      Remove_from_level(u);
      RecursiveDeref(T[u].low);
      RecursiveDeref(T[u].high);
   }
}
```

Problem 3. Given the procedure `ExchangeOrder(i)`, that will interchange the order of the variables i and $i+1$ (i is an index into the array `perm`), write the pseudo-code for `ExchangeOneNode(u,l1,l2)`, that will move node u from level $l1$ to $l2$.

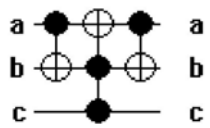
```
ExchangeOrder(i){
   int u = level[perm[i]];
   while(u != 0) {
      next = t[u].forward;
      ExchangeOneNode(u,perm[i],perm[i+1]);
      u = next;
      }
   }

ExchangeOneNode(u,l1,l2){
   low_child = T[u].low;
   high_child = T[u].high;
   if((T[low_child].i == l2) || (T[low_child].i == l2)){
      // at least one child must be at the next level
      if(T[low_child].i == l2){
         ll = T[low_child].low;
         lh = T[low_child].high;
      }
      else
         ll = lh = low_child;
      if(T[high_child].i == l2){
         hl = T[high_child].low;
         hh = T[high_child].high;
      }
      else
         hl = hh = high_child;
      Remove_from_level(u);
      T[u].i = l2;
```

```
        T[u].low = MK(l1,ll,hl);
        T[u].high = MK(l1,lh,hh);
        Add_to_level(u);
        RecursiveDeref(low_child);
        RecursiveDeref(high_child);
    }
}
```

Problem 4. Given the function $add4$ with 4 inputs and 3 outputs, where the inputs represent two 2-bit integers and the output represents their sum. How many garbage outputs are needed to make the function reversible? How many reversible functions (with the calculated number of variables) contain $add4$? Assume that the 3 output positions are fixed.

Problem 5. Find a Toffoli realization of the reversible function $F(a, b, c) = [3, 2, 1, 0, 4, 5, 6, 7]$. Use the second algorithm described in class.

Problem 6. Find the function realized by the following circuit.



Problem 7. Prove that there is no sequence of 3 Toffoli gates that produces the identity function.