Homework 3

Algorithmic Game Theory — Summer semester 2010

Note: there are 150 points on this homework, so it will be weighted more in your final mark than earlier homeworks.

Exercise 1 (45 points)**.** Consider the game in which there are two players and two piles of stones, with m stones in each pile. The players take turns in removing stones from the piles. On her turn, a player can remove as many stones as she wishes from one (and only one!) of the two piles. The player who forces her opponent to remove the last marble is the winner of the game.

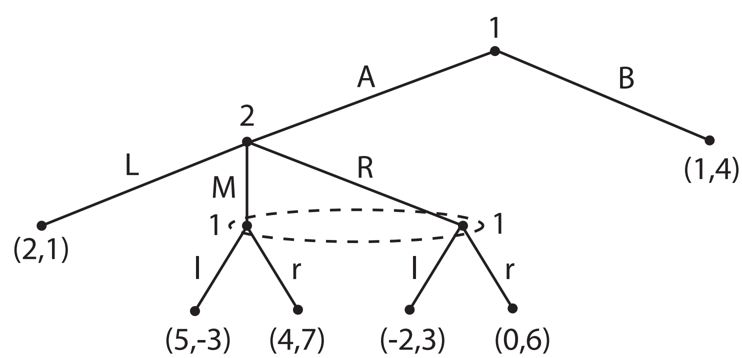
(a) Formalize the (2,2) version of this game as an extensive form game. (In order to reduce the size of the tree, you may use symmetry, e.g. removing a stone from the first pile is equivalent to removing a stone from the second pile.)

(b) Apply backward induction to the resulting game to find the subgame perfect equilibria. Using your result, determine which of the players has a winning strategy[1].

(c) Repeat the above for the (3,3) version of the game.

(d) Using the preceding analysis, determine which of the players has a winning strategy in the (m, m) variant (for each $m \geq 1$). Describe how the winning strategies work.

Exercise 2 (30 points)**.** Consider the following extensive form game.



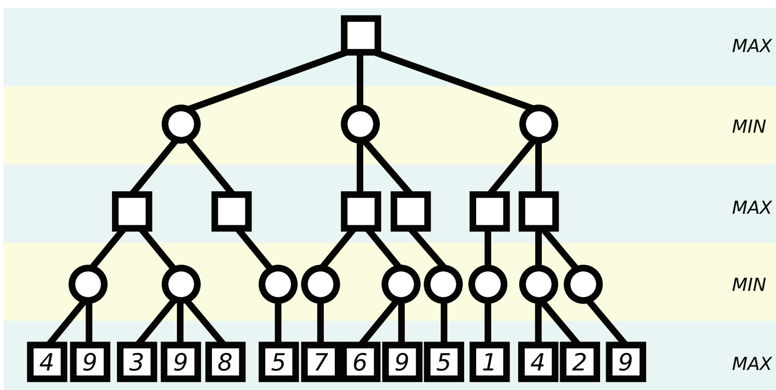
[1] We say a player has a *winning strategy* if he has a strategy which guarantees he will win no matter what strategy the other player adopts.

(a) Suppose that player 1 uses the mixed strategy giving probability 0.4 to pure strategy [B,r({AM,AR})], probability 0.1 to [B,l({AM,AR})], and probability 0.5 to [A,l({AM,AR})]. Let player 2 use the (mixed/behavioral) strategy giving probability p to L, q to M, and $1-p-q$ to R. Determine the probability of reaching each of the six leaf nodes.

(b) Now suppose $p = q = 0.3$. Determine the expected utilities of both players.

(c) Find a behavioral strategy that is equivalent to player 1's mixed strategy. Justify your response.

Exercise 3 (25 points)**.** Consider the following zero-sum extensive form game (numbers at leaf nodes give utility for Max player).



(a) Determine the value of each of the nodes using minimax.

(b) Determine which nodes will be visited (and which will be skipped) when using alpha-beta proceeding from left to right. Explain your answer.

(c) Repeat part (b) for the case where we proceed right to left.

Exercise 4 (20 points)**.** Consider the following two-player zero-sum strategic game.

1 \ 2	C	D	E
A	$(1, -1)$	$(-6, 6)$	$(4, -4)$
B	$(-2, 2)$	$(3, -3)$	$(1, -1)$

Create linear programs for the minmax strategies of both players. Using an online LP solver[2], compute the strategies of these players and the resulting utilities.

Exercise 5 (30 points)**.** Consider the following two-player strategic game.

1 \ 2	C	D
A	(-1, 7)	(6, 1)
B	(3, 3)	(2, 4)

(a) Create a linear program for the correlated equilibria of this game which maximize the sum of the players' utilities. Using an online LP solver, compute such a correlated equilibrium. What payoffs do the players get in this equilibrium?

(b) Now alter your linear program to compute the correlated equilibria in which the two players have the same expected utility. Give your linear program to an LP solver, and comment on the result.

[2]The one showed in class can be found at: `http://www.zweigmedia.com/RealWorld/simplex.html`.