

## Homework 5

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Algorithmic Game Theory

Summer semester 2010

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To get full marks, please make sure to justify your answers.

**Exercise 1** (15 points). Show that the median voting rule satisfies the Condorcet condition in the restricted setting of single-peaked preferences.

**Exercise 2** (10 points). Suppose we modify the Vickrey second-price auction by adding a reserve price. Formally, the seller chooses a price  $r$  and only sells the item if the highest bid is at least  $r$ . As before, the price the winner pays is the second-highest bid. Show that truthful bidding remains a dominant strategy for all players.

**Exercise 3** (10 points). Is truthful bidding a dominant strategy in third-price auctions? Explain your answer.

**Exercise 4** (25 points). Suppose there are three bidders for two objects  $(a, b)$ , and the bidders communicate the following valuations:

Bidder 1:  $\hat{v}_1(\emptyset) = 0, \hat{v}_1(\{a\}) = 3, \hat{v}_1(\{b\}) = 4, \hat{v}_1(\{a, b\}) = 11$

Bidder 2:  $\hat{v}_2(\emptyset) = 0, \hat{v}_2(\{a\}) = 5, \hat{v}_2(\{b\}) = 6, \hat{v}_2(\{a, b\}) = 6$

Bidder 3:  $\hat{v}_3(\emptyset) = 0, \hat{v}_3(\{a\}) = 7, \hat{v}_3(\{b\}) = 2, \hat{v}_3(\{a, b\}) = 10$

Compute the result of VCG mechanism (i.e. the allocation and payments).