Homework 2

Finite Automata on Infinite Words and Trees Winter semester, 2009-2010

Note: Graphical representations of automata are accepted.

Exercise 1 (25 points)**.** Consider the alphabet $\Sigma = \{0, 1\}$. Construct Büchi automata which accept each of the following languages:

1. $\{w \,|\,$ the symbol 0 appears in w exactly twice$\}$
2. $\{w \,|\,$ every 0 which appears in w is followed immediately by 11$\}$
3. $\{w \,|\, w$ does not contain the substring 000 $\}$
4. $\{w \,|\, w$ contains finitely many substrings 11 $\}$
5. $\{w \,|\, w$ contains finitely many substrings 11 but infinitely many 1's $\}$

Exercise 2 (25 points)**.** Prove or disprove the following two statements:

1. The set of languages recognized by deterministic Büchi automata is closed under union.
2. The set of languages recognized by deterministic Büchi automata is closed under intersection.

Exercise 3 (25 points)**.** Suppose that L is a Büchi recognizable language over the alphabet

$$\Sigma = \Sigma_1 \times \Sigma_2 = \{(a, b) \,|\, a \in \Sigma_1, b \in \Sigma_2\}$$

Prove that the languages $\textsc{Pr}_1(L)$ and $\textsc{Pr}_2(L)$ defined by

$$\textsc{Pr}_1 = \{u \in \Sigma_1^\omega \,|\, \exists v \in \Sigma_2^\omega \text{ such that } (u[0], v[0])(u[1], v[1])(u[2], v[2]) \ldots \in L\}$$
$$\textsc{Pr}_2 = \{v \in \Sigma_2^\omega \,|\, \exists u \in \Sigma_1^\omega \text{ such that } (u[0], v[0])(u[1], v[1])(u[2], v[2]) \ldots \in L\}$$

are both Büchi recognizable.

Exercise 4 (25 points)**.** Given a Büchi recognizable language $\textsc{Select}$ over alphabet $\{1, 2\}$ and two Büchi recognizable languages L_1, L_2 over alphabet $\{a, b\}$, prove that the following language $\textsc{Fusion}$ is also Büchi recognizable:

$$\textsc{Fusion} = \{\sigma \,|\, \text{there exists } \alpha \in L_1 \text{ and } \beta \in L_2 \text{ and } \gamma \in \textsc{Select} \text{ such that } \sigma[i] = \alpha[i] \text{ if } \gamma[i] = 1, \text{ and } \sigma[i] = \beta[i] \text{ if } \gamma[i] = 2\}$$