Homework 5

Finite Automata on Infinite Words and Trees Winter semester, 2009-2010

Exercise 1 (40 points)**.** Let $\Sigma = \{f/2, g/1, a/0, b/0\}$. Define non-deterministic top-down and deterministic bottom-up tree automata which recognize the following tree languages:

1. the set of all trees of even height which do not contain f
2. the set of all trees which contain both a and b
3. the set of all trees t such that $t(\lambda) = f$, $t(1) = t(2) = g$
4. the set of all trees which contain a subtree of the form $f(a, b)$

Are there deterministic top-down automata which accept the above languages? Either give such an automaton or explain why there cannot be such an automaton.

Exercise 2 (20 points)**.** Let $L_n = \{or/2, and/2, not/1, x_1/0, ..., x_n/0\}$. A L_n-tree can be viewed as a Boolean formula over the variables $x_1, \ldots, x_n$. Define a DFTA which recognizes the set of satisfiable Boolean formulae over $x_1, \ldots, x_n$ (i.e. those which are true under some assignment of values to the variables).

Exercise 3 (20 points)**.** Let $\Sigma = \{f/2, a/0, b/0\}$. Use the pumping lemma to show that the following tree languages cannot be recognized by a NFTA:

1. the set of all trees t such that $t(\lambda) = f$ and $t_1 = t_2$
2. the set of all trees which have the same number of a's as b's

Exercise 4 (20 points)**.** This exercise concerns the closure properties of languages recognized by deterministic top-down tree automata.

1. Show the set of languages accepted by deterministic top-down tree automata is closed under intersection.

2. Show the set of languages accepted by deterministic top-down tree automata is not closed under union. *Hint:* consider languages of trees in which all leaf nodes have the same symbol.

3. Use parts 1 and 2 to show that the set of languages accepted by deterministic top-down tree automata is not closed under complementation.