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From syllogism to common sense ...

Exercise Sheet 3: Propositional Logic

To be discussed on 1 December 2011

1. Assume that $\bot, \top$ are defined by $\bot = (p \wedge \neg p)$ and $\top = \neg\bot$. Show that $w\top = 1$ and $w\bot = 0$ for all valuations w.

2. Show the following equivalences via truth tables.

   a) the de Morgan rules $\neg(\alpha \wedge \beta) \equiv (\neg\alpha \vee \neg\beta)$ and $\neg(\alpha \vee \beta) \equiv (\neg\alpha \wedge \neg\beta)$

   b) the rule that allows arbitrary order of premises in a chain of two implications:

   $$\alpha \to \beta \to \gamma \;\equiv\; \alpha \wedge \beta \to \gamma \;\equiv\; \beta \to \alpha \to \gamma$$

   c) the formulas from the "strange natural language example":

   $$(S \to H) \wedge (P \to H) \;\equiv\; (S \vee P) \to H$$

3. Prove that the signature $\{\uparrow\}$ is functionally complete.

4. Prove the replacement theorem.

5. *(*) Only if you enjoy proving theorems.*

   Prove the unique formula reconstruction property. Proceed in two steps, the first of which is almost immediate.

   a) *Verify that a compound formula φ is either of the form $\varphi = \neg\alpha$ or $\varphi = (\alpha \wedge \beta)$ or $\varphi = (\alpha \vee \beta)$ for suitable formulas α, β.*

   b) *Prove the following proposition. A proper initial segment of a formula is never a formula. Equivalently: if $\alpha\xi = \beta\eta$ for formulas α, β and arbitrary strings ξ, η, then $\alpha = \beta$.*

   With the help of this proposition, prove the claim of uniqueness in the theorem.