From syllogism to common sense: a tour through the logical landscape

Mehul Bhatt Oliver Kutz Thomas Schneider

Introduction 3 November 2011

And now . . .

When, where, how?

- Lecture Thursdays 16–18, MZH 1110
- Tutorial Thursdays 18–19, MZH 1110
- Room change: 3 Nov and 17 Nov in MZH 3150
- ECTS: 4
- Literature will be announced as we go along
- Lecture homepage: `tinyurl.com/ws2011-logic`

Exam

At the end of the semester, pick 2 topics from the course, present them to us in an oral exam and answer our questions.

Who?

Mehul Bhatt  bhatt

Oliver Kutz  okutz

Thomas Schneider  tschneider

@informatik.uni-bremen.de

And now . . .

Aristotle's Syllogisms

- First formal study of logic in Aristotle's *Prior Analytics*
- Central elements: schemes such as

If all A are B		If all men are mortal
and all B are C	e.g.	and all Greeks are men
then all A are C		then all Greeks are mortal

- We will
  - explain and discuss valid types of syllogism,
  - show their corresponding Venn diagrammes,
  - explore common fallacies (reasoning mistakes with syllogisms),
  - play with the Interactive Syllogistic Machine

Propositional logic

- The most basic classical logic: allows to combine atomic propositions using Boolean operators ("and", "or", "not")
- Two notions of consequence/entailment: proof-theoretic ($\vdash$) and semantic ($\models$)

  $\vdash$ implies $\models$: soundness of the proof theory

  $\models$ implies $\vdash$: completeness

- We'll introduce basic terminology
- We'll also look at conditionals such as

  If he takes a plane he will get here quicker.
  He will take a plane.
  Hence, he will get here quicker.

  If New York is in New Zealand, then $2 + 4 = 4$.

Intuitionistic logic

- Brouwer's Intuitionistic logic developed out of the crisis in the foundations of mathematics in the late 19th century
- The philosophy of intuitionism puts a stong emphasis on the notion of *(mental) construction*. Its logic therefore
  - rejects the classical principle of tertium non datur (excluded middle) and double negation elimination
  - it is a *restriction* of classical logic
  - not truth is preserved, but *justification*
  - proof by contradiction can not be used, as it does not yield a *construction*

We will look at

- Heyting's axiomatisation of intuitionistic propositional logic
- Kripke frames as a semantics for intuitionistic logic in terms of 'stages of knowledge'
- the Gödel translation establishing a correspondence between classical and intuitionistic logic

Modal logic

- *Extends* propositional logic
- Concerned with the *modes* in which things can be true/false:

  φ is *necessary* or *possible*

  φ is true *sometimes in the future* or *always in the future*

  the agent *knows* or *believes* φ

  φ is *provable*

- Different axiomatisations lead to different modal systems
- Possible-world semantics
- Modal tableaux
- Soundness, completeness
- Strict conditional and non-normal modal logics

Many-valued logic

- Many-valued logics give up the idea that there are exactly 2 truth-values
- They evaluate formulae not only to true or false, but e.g. to:
  - true, false, or *indeterminate* (**Łukasiewicz**)
  - true, false, *neither true nor false*, *both true and false* (**Belnap**)
  - true, false, degrees of truth in the interval $[0, \ldots, 1]$ (**Zadeh**)
  - etc.
- Such logics need more abstract model theory than Boolean Algebras
- Many applicaton areas, such as database theory (SQL), uncertain reasoning, and paraconsistent reasoning
- We discuss basics of model theory, truth tables, axiomatisations, and the relation to classical logic

Description logic

- Allows for describing terminological knowlegde, e.g.
  - $\text{Elephant} \equiv \text{Mammal} \sqcap \exists \text{hasBodypart.Trunk} \sqcap \forall \text{hasColour.Grey}$ defines elephants extensionally
  - $$\text{Fraction} \sqcap \exists \text{hasLocation.Arm} \sqsubseteq \exists \text{hasLocation.}(\text{Ulna} \sqcup \text{Radius} \sqcup \text{Humerus})$$ states background knowledge about bone fractions
- DLs are restricted, well-behaved fragments of first-order logic
- DL theories *(ontologies)* are used in biomedicine, healthcare, linguistics, physics, chemistry, engineering, Semantic Web, . . .
- Sophisticated tools can automatically infer implicit knowledge (reasoning), explain inferences, extract modules for reusing ontologies, integrate ontologies and databases, . . .

First-order logic

- Extend propositional logic:
  predicates, functions, Boolean operators, quantification
- Used to formalise axiomatic systems
  and other fundamental mathematical knowledge, e.g.
  - $\forall xyz(xRy \wedge yRz \rightarrow xRz)$: relation R is transitive
  - $(\forall n\, P(n)) \leftrightarrow \Big(P(0) \wedge \forall n\, \big(P(n) \rightarrow P(n+1)\big)\Big)$: induction
  - Zermelo-Fraenkel set theory, Peano arithmetic, ...
- Also used as an ontology language more expressive than DLs
- Enjoys important meta-logical properties, but
  is undecidable $\rightsquigarrow$ not as computationally well-behaved as DLs
- We'll cover syntax, semantics, deduction system(s),
  completeness, compactness, Löwenheim-Skolem theorem,
  first-order ontologies, tool demo

Commonsense reasoning

...

. . .