

From syllogism to common sense: a tour through the logical landscape

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Introduction 3 November 2011

When, where, how?

- Lecture Thursdays 16–18, MZH 1110
- Tutorial Thursdays 18–19, MZH 1110
- Room change: 3 Nov and 17 Nov in MZH 3150
- ECTS: 4
- Literature will be announced as we go along
- Lecture homepage: tinyurl.com/ws2011-logic

Exam

At the end of the semester, pick 2 topics from the course, present them to us in an oral exam and answer our questions.

And now ...

1 Organisational aspects

2 Topics

Who?

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And now ...

1 Organisational aspects

2 Topics

Propositional logic

- The most basic classical logic: allows to combine atomic propositions using Boolean operators ("and", "or", "not")
- Two notions of consequence/entailment: proof-theoretic (\vdash) and semantic (\models)
 - \vdash implies \models : soundness of the proof theory
 - \models implies \vdash : completeness
- We'll introduce basic terminology
- We'll also look at conditionals such as
 - If he takes a plane he will get here quicker.
 - He will take a plane.
 - Hence, he will get here quicker.
- If New York is in New Zealand, then $2 + 4 = 4$.

Aristotle's Syllogisms

- First formal study of logic in Aristotle's *Prior Analytics*
- Central elements: schemes such as

If all A are B and all B are C then all A are C	e.g.	If all men are mortal and all Greeks are men then all Greeks are mortal
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- We will
 - explain and discuss valid types of syllogism,
 - show their corresponding Venn diagrammes,
 - explore common fallacies (reasoning mistakes with syllogisms),
 - play with the Interactive Syllogistic Machine

Intuitionistic logic

- Brouwer's Intuitionistic logic developed out of the crisis in the foundations of mathematics in the late 19th century
- The philosophy of intuitionism puts a strong emphasis on the notion of (*mental*) *construction*. Its logic therefore
 - rejects the classical principle of tertium non datur (excluded middle) and double negation elimination
 - it is a *restriction* of classical logic
 - not truth is preserved, but *justification*
 - proof by contradiction can not be used, as it does not yield a *construction*
- We will look at
 - Heyting's axiomatisation of intuitionistic propositional logic
 - Kripke frames as a semantics for intuitionistic logic in terms of 'stages of knowledge'
 - the Gödel translation establishing a correspondence between classical and intuitionistic logic

Modal logic

- Extends propositional logic
- Concerned with the *modes* in which things can be true/false:
 - φ is *necessary* or *possible*
 - φ is true *sometimes in the future* or *always in the future*
 - the agent *knows* or *believes* φ
 - φ is *provable*
- Different axiomatisations lead to different modal systems
- Possible-world semantics
- Modal tableaux
- Soundness, completeness
- Strict conditional and non-normal modal logics

Description logic

- Allows for describing terminological knowledge, e.g.
 - Elephant \equiv Mammal \sqcap \exists hasBodypart.Trunk \sqcap \forall hasColour.Grey defines elephants extensionally
 - Fraction \sqcap \exists hasLocation.Arm \sqsubseteq \exists hasLocation.(Ulna \sqcup Radius \sqcup Humerus) states background knowledge about bone fractions
- DLs are restricted, well-behaved fragments of first-order logic
- DL theories (*ontologies*) are used in biomedicine, healthcare, linguistics, physics, chemistry, engineering, Semantic Web, ...
- Sophisticated tools can automatically infer implicit knowledge (reasoning), explain inferences, extract modules for reusing ontologies, integrate ontologies and databases, ...

Many-valued logic

- Many-valued logics give up the idea that there are exactly 2 truth-values
- They evaluate formulae not only to true or false, but e.g. to:
 - true, false, or *indeterminate* (**Łukasiewicz**)
 - true, false, *neither true nor false*, *both true and false* (**Belnap**)
 - true, false, degrees of truth in the interval $[0, \dots, 1]$ (**Zadeh**)
 - etc.
- Such logics need more abstract model theory than Boolean Algebras
- Many application areas, such as database theory (SQL), uncertain reasoning, and paraconsistent reasoning
- We discuss basics of model theory, truth tables, axiomatisations, and the relation to classical logic

First-order logic

- Extend propositional logic: predicates, functions, Boolean operators, quantification
- Used to formalise axiomatic systems and other fundamental mathematical knowledge, e.g.
 - $\forall xyz(xRy \wedge yRz \rightarrow xRz)$: relation R is transitive
 - $(\forall n P(n)) \leftrightarrow (P(0) \wedge \forall n (P(n) \rightarrow P(n+1)))$: induction
 - Zermelo-Fraenkel set theory, Peano arithmetic, ...
- Also used as an ontology language more expressive than DLs
- Enjoys important meta-logical properties, but is undecidable \leadsto not as computationally well-behaved as DLs
- We'll cover syntax, semantics, deduction system(s), completeness, compactness, Löwenheim-Skolem theorem, first-order ontologies, tool demo