From syllogism to common sense:
a tour through the logical landscape

Categorical syllogisms

Mehul Bhatt Oliver Kutz *Thomas Schneider*

10 November 2011

Why are we dealing with syllogism today?

- Syllogism goes back to ARISTOTLE:
  (ARISTOTLE: 384–322 BCE, Greek/Macedonian philosopher, "The philosopher", founder of logic)
- Want to explore history *and* foundations of modern logic

Do ask whenever you have questions regarding content or language.

Towards a theory of deduction

- Deduction = drawing of inferences (logical conclusions) from a set of statements
- Focus here on deductive arguments (DAs) such as:
  "If A is true and B is true, then C is true"
  A, B: premises, C: conclusions
- Goal: analyse structure of DAs, determine when a DA is *valid*
- In general: DA is valid if "its premise(s) assure the truth of the conclusion with necessity"
  i.e., premise(s) can't be true without the conclusion being true
- Theory of deduction explains relations between premises and conclusion in valid arguments

Part I

Categorical propositions (CPs)

In this part . . .

And now . . .

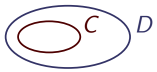
Classes and relations

Class of objects:

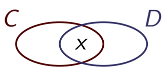
Collection of objects with common characteristics – a set!

Relations between classes

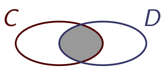
- C wholly included in D $\quad C \subseteq D$
  e.g. C: all dogs, D: all mammals



- C partially included in D $\quad C \cap D \neq \emptyset$
  e.g. C: all athletes, D: all females



- C, D have no members in common $\quad C \cap D = \emptyset$
  (mutual exclusion, disjointness)
  e.g. C: all triangles, D: all circles



Categorical propositions and deductive arguments

Categorical proposition (CP): states relations between classes

Deductive argument: sequence of CPs

Example:

No athletes are vegetarians.
All football players are athletes.
Therefore no football players are vegetarians.

And now . . .

The four standard forms for categorical propositions

Example	General form	Name
All policitians are liars.	All S is P.	**A** Universal affirmation
No policitians are liars.	No S is P.	**E** Universal negation
Some policitians are liars.	Some S is P.	**I** Particular affirmation
Some policitians are not liars.	Some S is not P.	**O** Particular negation

General remarks

- S: subject class
- P: predicate class
- "is" can be replaced by any form of the verb "be":
  is, are, was, were, will be, . . .

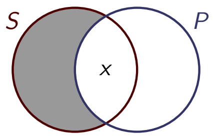
A Universal affirmation

All S is P.

I.e., the whole of S is included in P.

A-propositions *affirm* the relation of class inclusion *universally*.

Representation as Venn diagramme
(John Venn, 1834–1923, English mathematician/logician, Hull)



This traditional interpretation assumes that S is not empty.

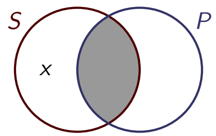
E Universal negation

No S is P.

I.e., the whole of S is excluded from P.

E-propositions *deny* the relation of class inclusion *universally*.

Representation as Venn diagramme:



Note: this is *not* symmetric in S, P in the traditional interpretation.

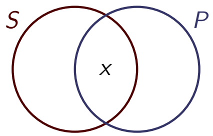
I Particular affirmation

Some S is P.

I.e., S and P have at least one member in common.

I-propositions *affirm* the relation of class intersection *partially*, for some *particular* member.

Representation as Venn diagramme:



Note: this is symmetric in S, P!

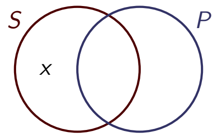
O Particular negation

Some S is not P.

I.e., At least one member of S is excluded from P.

I-propositions *deny* the relation of class intersection *partially*, for some *particular* member.

Representation as Venn diagramme:



Summary

- **A**-, **E**-, **I**-, and **O**-propositions are the building blocks of deductive arguments.
- *S*ubject and *P*redicate can be single nouns or more intricate descriptions of classes of objects:
  - All politicians are liars.
  - No ancient Greeks were computer scientists.
  - Some candidates for the position are persons of honour and integrity.
  - Some of today's technology will not be of eternal life.
- Aristotle's analysis of these building blocks and their interplay lay the foundation for today's systems for analysis of deductive arguments.

Examples and exercises

Identify subject and predicate terms.

- No athletes who have ever accepted pay for participating in sports are amateurs.
- All satellites that are currently in orbit less than 1000 miles high are very delicate devices that cost many thousands of euros to manufacture.
- Some historians are extremely gifted writers whose works read like first-rate novels.

And now . . .

Quality and quantity

. . . lie in the names of the four types:

Quality / Quantity	affirmative	negative
universal	All S is P. Universal affirmation	No S is P. Universal negation
particular	Some S is P. Particular affirmation	Some S is not P. Particular negation

⇝ General schema for propositions:

Quantifier	(subject term)	copula	(predicate term)
All		is (not)	
No		are (not)	
Some		were (not)	
⋮		will (not) be	
		⋮	

Distribution

Does a proposition make a statement about all members of S or P?

- **A** All S is P.
  S is distributed: all members of S are included in P
  P is not
- **E** No S is P.
  S is distributed: all members of S are excluded from P
  P is distributed: all members of P are excluded from S
- **I** Some S is P.
  Neither S nor P are distributed.
- **O** Some S is not P.
  P is distr.: all members of P excluded from a *part of* S
  S is not

Overview

Proposition		Quantity	Quality	Distributes
All S is P.	**A**	Universal	Affirmative	S
No S is P.	**E**	Universal	Negative	S, P
Some S is P.	**I**	Particular	Affirmative	—
Some S is not P.	**O**	Particular	Negative	P

- (Negative propositions distribute their predicate term; positive propositions don't.)
- (Universal propositions distribute their subject term; particular propositions don't.)

Examples and exercises

Identify quality, quantity, and contribution.

- Some presidential candidates will be sadly disappointed people.
- All those who died in Nazi concentration camps were victims of a cruel and irrational tyranny.
- Some recently identified unstable elements were not entirely accidental discoveries.

And now . . .

Contradictories and contraries

Contradictories are two props that are negations of each other, i.e., exactly one is true.

- **A** and **O**
- **E** and **I**

Contraries are 2 non-contradictory props that cannot both be true, i.e., if one is true, then the other is false.

(“Werder will win the next game.” $\leftrightarrow$ “Werder will lose the next game.”)

- **A** and **E** – only traditionally

Subcontraries and Subalternation

Subcontraries: 2 non-contradictory props that cannot both be false, i.e., if one is false, then the other is true.

(“I’m at least as smart as you.” $\leftrightarrow$ “I’m at most as smart as you.”)

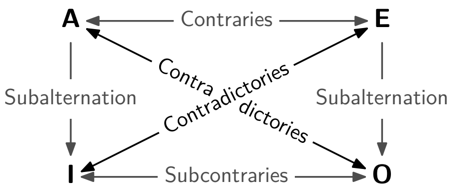
- **I** and **O** – only traditionally

A subaltern of a univ. CP is the partic. CP of the same quality, i.e., the univ. CP implies the partic. CP (only traditionally).

- **I** is the subalternate of **A**
- **O** is the subalternate of **E**

The square of opposition

All four relations in one diagramme:



Can be used to draw

- immediate inferences

  If **A** true, then **E** false, **I** true, **O** false.
  If **A** false, then **O** true. (**E**, **I** undetermined)
  If **I** true, then **E** false. (**A**, **O** undetermined)
  . . .

- mediate inferences from a *set* of premises: syllogisms . . .

Examples and exercises

Of the following sets of 4 propositions,

1. assume that the first is true.
2. assume that the first is false.

What can we conclude about the truth or falsehood of the others?

All successful executives are intelligent people.
No successful executives are intelligent people.
Some successful executives are intelligent people.
Some successful executives are not intelligent people.

Some college professors are not entertaining lecturers.
All college professors are entertaining lecturers.
No college professors are entertaining lecturers.
Some college professors are entertaining lecturers.

Further immediate inferences

. . . are drawn without using the square of opposition

Conversion exchanges subject and predicate term in a CP.
Is only successful if the premise (traditionally) implies the conclusion.

Premise	Successful?	Conclusion
A All *S* is *P*.	through limitation: **I** Some *S* is *P*.	**I** Some *P* is *S*.
E No *S* is *P*.	yes	**E** No *P* is *S*.
I Some *S* is *P*.	yes	**I** Some *P* is *S*.
O Some *S* is not *P*.	no	

(Draw the Venn diagramme to see
why the direct conversion of **A** and **O** is not successful.)

Further immediate inferences

Complements of *S* and *P* are "non-*P*" and "non-*S*"
Think of set complements. Clearly, non-non-$X = X$.

Do not mistake complements with complementary terms:

non-hero $\neq$ coward
non-elephant $\neq$ mouse
non-lecturer $\neq$ student

Obversion leads to a logically equivalent CP
by changing the quality and replacing *P* with non-*P*.

Premise	Conclusion
A All *S* is *P*.	**E** No *S* is non-*P*.
E No *S* is *P*.	**A** All *S* is non-*P*.
I Some *S* is *P*.	**O** Some *S* is non non-*P*.
O Some *S* is not *P*.	**I** Some *S* is non-*P*.

Further immediate inferences

Contraposition replaces S with non-P, and P with non-S

Premise	Successful?	Conclusion
A All S is P.	yes	**A** All non-P is non-S.
E No S is P.	through limitation: **O** Some S is not P.	**O** Some non-P is not non-S.
I Some S is P.	no	
O Some S is not P.	yes	**O** Some non-P is not non-S.

(Draw the Venn diagramme to see
why the direct contraposition of **E** and **I** is not successful.)

And now . . .

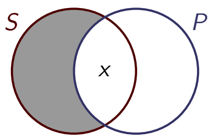
Aristotelian interpretation and existential import

Aristotelian interpretation (until now called "traditional") assumes:
whenever we speak of the subject class S, it cannot be empty.

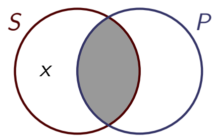
$\rightsquigarrow$ **A** All S is P. implies **I** Some S is P.
E No S is P. implies **O** Some S is not P.

Propositions made under this assumption have existential import.

Aristotelian interpretation in Venn diagrammes:



A



E

Boolean interpretation

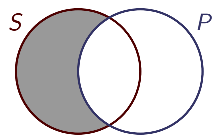
(George Boole, 1815–64, English mathematician, Cork/Ireland)

Boolean interpretation rejects existential import.

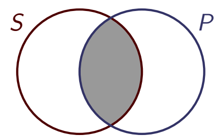
A All S is P means: If there is such a thing as an S, it is always in P.

E No S is P means: If there is such a thing as an S, it is not in P.

Boolean interpretation in Venn diagrammes:



A

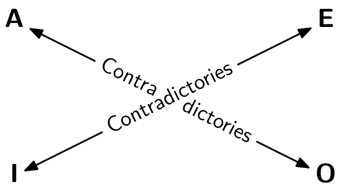


E

Modern logic adopts Boolean interpretation.
Arguments relying on exist. import commit the existential fallacy.

The "modern" square of opposition

. . . admits less inferences:



And now . . .

Summary

Categorical propositions

- relate classes of objects
- come in four types **A**, **E**, **I**, **O**
- have a quality, quantity, and distribution
- can be related in the square of opposition
- can be used to draw simple inferences
- can be interpreted with existential import (Aristotelian) or without (Boolean)

From now on, we'll interpret CPs in the Boolean way.

Part II

Categorical syllogisms (CSs)

In this part . . .

And now . . .

Basic notions

Aim: more extended reasoning with CPs

Syllogism: deductive argument with 2 premises and 1 conclusion

Categorical syllogism:

- syllogism based on CPs
- deductive argument of 3 CPs
- all 3 CPs together contain 3 terms
- every term occurs in 2 propositions

Syllogisms are common, clear and easily testable. They are

> *one of the most beautiful and also one of the most important made by the human mind.*

(Gottfried Wilhelm Leibniz, 1646–1716, German philosopher and mathematician, Hannover)

Standard-form CSs

1. Premises and conclusion are standard CPs (**A**, **E**, **I**, **O**)
2. CPs are arranged in standard order:

$$\begin{array}{r} \dots S_1 \text{ is } \dots P_1 \\ \dots S_2 \text{ is } \dots P_2 \\ \hline \therefore\ \dots S \text{ is } \dots P \end{array}$$

P: major term, S: minor term

Remember: 3 terms altogether, each in 2 propositions!
$\rightsquigarrow$ S_1, S_2, P_1, P_2 consist of P, S and a third term: the middle term

Major premise contains P, M
Minor premise contains S, M

Examples

Major term, minor term, middle term

All great scientists are college graduates.
Some professional athletes are college graduates.
Therefore some professional athletes are great scientists.

All artists are egotists.
Some artists are paupers.
Therefore some paupers are egotists.

Mood of a CS

Mood of a CS is the pattern of types of its three CPs,
in the order major premise – minor premise – conclusions

A	All artists are egotists.	
I	Some artists are paupers.	Mood **AII**
I	Therefore some paupers are egotists.	

$\rightsquigarrow$ $4^3 = 64$ moods

Figure of a CS

Figure of a CS: combination of order of S, M, P in the premises:

$$\begin{array}{ll} & \text{No } P \text{ is } M \\ & \text{Some } S \text{ is not } M \\ \hline \therefore & \text{All } S \text{ is } P \end{array} \quad \text{has figure} \quad \begin{array}{l} P\text{–}M \\ S\text{–}M \\ \hline \therefore S\text{–}P \end{array}$$

$\rightsquigarrow$ 4 figures:

$$(1)\ \begin{array}{l} M\text{–}P \\ S\text{–}M \\ \hline \therefore S\text{–}P \end{array} \quad (2)\ \begin{array}{l} P\text{–}M \\ S\text{–}M \\ \hline \therefore S\text{–}P \end{array} \quad (3)\ \begin{array}{l} M\text{–}P \\ M\text{–}S \\ \hline \therefore S\text{–}P \end{array} \quad (4)\ \begin{array}{l} P\text{–}M \\ M\text{–}S \\ \hline \therefore S\text{–}P \end{array}$$

Formal nature of the syllogistic argument

There are only $4 \cdot 64 = 256$ possible *forms* of CSs.

Their validity can be exhaustively analysed and established.

Only a few will turn out to be valid.

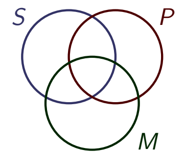
Infinitely many (in-)valid syllogistic arguments can be obtained by replacing S, M, P in a(n in-)valid CS with "real-world" class descriptions.

And now . . .

Testing a form of CS for validity

. . . is very simple!

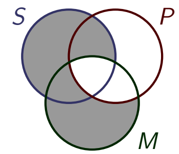
1. Draw three overlapping cycles for S, P, M:



2. Mark the premises according to their types as earlier.

   E.g.: **AAA**-1

   All M is P.
   All S is M.
   ∴ All S is P.



3. Try to read off the conclusion without further marking.
   Syllogism type is valid iff reading off was successful.

Example

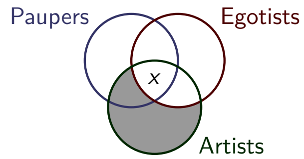
What form does this syllogism have? Is it valid?

All dogs are mammals.
All cats are mammals.
Therefore all cats are dogs.

Two cautions

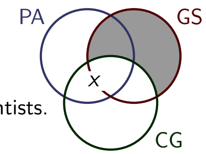
(1) Mark universal before particular premise.

All artists are egotists.
Some artists are paupers.
Therefore some paupers are egotists.



(2) If a particular premise speaks about two nonempty regions, put the x on the boundary of these regions.

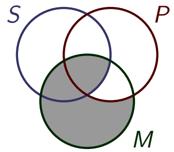
All great scientists are college graduates.
Some professional athletes are college graduates.
Therefore some professional athletes are great scientists.



Examples

AEE-1

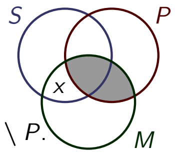
All M is P.
No S is M.
∴ No S is P.



Invalid: diagramme does not exclude S from P.

EIO-4

No P is M.
Some M is S.
∴ Some S is not P.



Valid: diagramme gives a particular instance of $S \setminus P$.

And now . . .

An alternative characterisation of validity of CSs

. . . via rules that focus on the *form* of the syllogism

Rule 1: Avoid four terms.

- With $\geqslant 4$ terms, it's no syllogism at all
- Beware of equivocations!
  (two occurrences of the same word with different meanings)

  And the Lord spake, saying, "First shalt thou take out the Holy Pin. Then, shalt thou count to three. No more. No less. Three shalt be the number thou shalt count, and the number of the counting shall be three. Four shalt thou not count, neither count thou two, excepting that thou then proceed to three. Five is right out. Once at the number three, being the third number to be reached, then, lobbest thou thy Holy Hand Grenade of Antioch towards thy foe, who, being naughty in My sight, shall snuff it."

  (from "Monty Python and the Holy Grail", 1975)

Distribute your middle term

Rule 2: Distribute the middle term in at least one premise.

(One proposition must refer to *all* members of M.)

Example: All Russians were revolutionists.
All anarchists were revolutionists.
Therefore all anarchists were Russians.

Fallacy: middle term "revolutionists" doesn't link S, P

- Russians are included in a part of revolutionists
- Anarchists are included in a part of revolutionists, possibly a *different part!*

Fallacy of the undistributed middle

Watch your distribution

Rule 3: Any term distributed in the conclusion must be distributed in the premises.

Intuition: if premises speak about *some* members of a class, we cannot conclude anything about *all* members of that class.

Example: All dogs are mammals.
No cats are dogs.
Therefore no cats are mammals.

Fallacy: "mammals" is distributed in the conclusion, but not in the major premise.

Fallacy of illicit process (here: illict process of the major term)

Two negative premises are bad

Rule 4: Avoid two negative premises.

- 2 negative premises
  $\rightsquigarrow$ 2$\times$ class exclusion between S, M and between P, M
- No power to enforce any relation between S, P
- Try all nine possibilities in a Venn diagramme!

Fallacy of exclusive premises

Don't turn neg into pos; don't be so Aristotelian

Rule 5: If $\geqslant 1$ premise is negative, the conclusion must be neg.

Example: No poets are accountants.
Some artists are poets.
Therefore some artists are accountants.

Fallacy of drawing an affirmative conclusion from a neg. premise

Rule 6: From two universal premises, no particular conclusion may be drawn.

Example: All household pets are domestic animals.
No unicorns are domestic animals.
Therefore some unicorns are not household pets.

Existential fallacy
(which is not a fallacy in the Aristotelian interpretation)

And now ...

The 15 valid forms of syllogisms

AAA-1	Barbara	**AII**-3	Datisi
EAE-1	Celarent	**IAI**-3	Disamis
AII-1	Darii	**EIO**-3	Ferison
EIO-1	Ferio	**OAO**-3	Bokardo
AEE-2	Camestres	**AEE**-4	Camenes
EAE-2	Cesare	**IAI**-4	Dimaris
AOO-2	Baroko	**EIO**-4	Fresison
EIO-2	Festino		

And now . . .

Summary

Categorical syllogisms . . .

- are deductive arguments consisting of 3 CPs
- require a certain amount of interaction between the terms in their CPs
- come in 4 figures and 64 moods
- can be tested for validity using Venn diagrammes or rules/fallacies

There are 15 valid forms of syllogisms in Boolean interpretation, 24 in Aristotelian interpretation

It's almost play time:
`http://www.theotherscience.com/syllogism-machine`

Try with examples from Pages 41 47 53 55

Literature and outlook

Contents is taken from Chapters 5, 6 of

I. Copi, C. Cohen, K. McMahon:
Introduction to Logic, 14th ed., Prentice Hall, 2011.

SUUB Magazin 02 E 2115
Link to available copies

Chapter 7:
transform arguments of everyday speech into syllogistic form, possible difficulties

Literature and outlook

Contents is taken from Chapters 5, 6 of

I. Copi, C. Cohen, K. McMahon:
Introduction to Logic, 14th ed., Prentice Hall, 2011.

SUUB Magazin 02 E 2115
Link to available copies

Chapter 7:
transform arguments of everyday speech into syllogistic form,
possible difficulties

Thank you.