Lightweight Temporal Description Logics with Rigid Roles and Restricted TBoxes

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Motivation

Vision: Express **temporal** knowledge in a DL ontology

Applications: KR & reasoning …

… over temporal conceptual data models

… in the medical domain

Example: ‘A patient who has diabetes **now** may develop certain disorders **in the future**’

Approach: Extend DLs with point-based temporal operators

$\rightsquigarrow$ **Temporal description logics (TDLs)** [Schild ’93]

The TDL landscape

Previous work

$\mathcal{ALC}/\mathcal{EL}$/DL-Lite + LTL/CTL $\rightsquigarrow$ PTIME . . . undecidable

[Artale et al. 2007/14, Baader et al. 2008, Gutiérrez-Basulto et al. 2012/14]

Challenges

- Allow **rigid roles** to capture time-invariant relations

  e.g.: hasBloodGroup, hasGeneticDisease, …

- With rigid roles, even $\mathcal{EL}$+$\Diamond$ and $\mathcal{EL}$+E$\bigcirc$ are **undecidable!**

Goal

Indentify decidable (and tractable) fragments of $\mathcal{EL}$+CTL

TDLs in a nutshell: syntax

TDLs are modal description logics – here $\mathcal{EL}$ + CTL:

$$C := \underbrace{\top \mid A \mid C \sqcap C \mid \exists r.C}_{\mathcal{EL}} \mid \underbrace{\mathsf{E}\Diamond C \mid \mathsf{E}\bigcirc C \mid \mathsf{A}\Box C \mid \ldots}_{\text{CTL operators}}$$

$\mathcal{ALC}$ + CTL additionally allows $\neg$ (and thus $\sqcup$, $\forall$)

Example: $\exists\, \text{hasDisease.Diabetes} \sqsubseteq \mathsf{E}\Diamond\, \exists\, \text{hasDisease.Glaucoma}$

Design choices

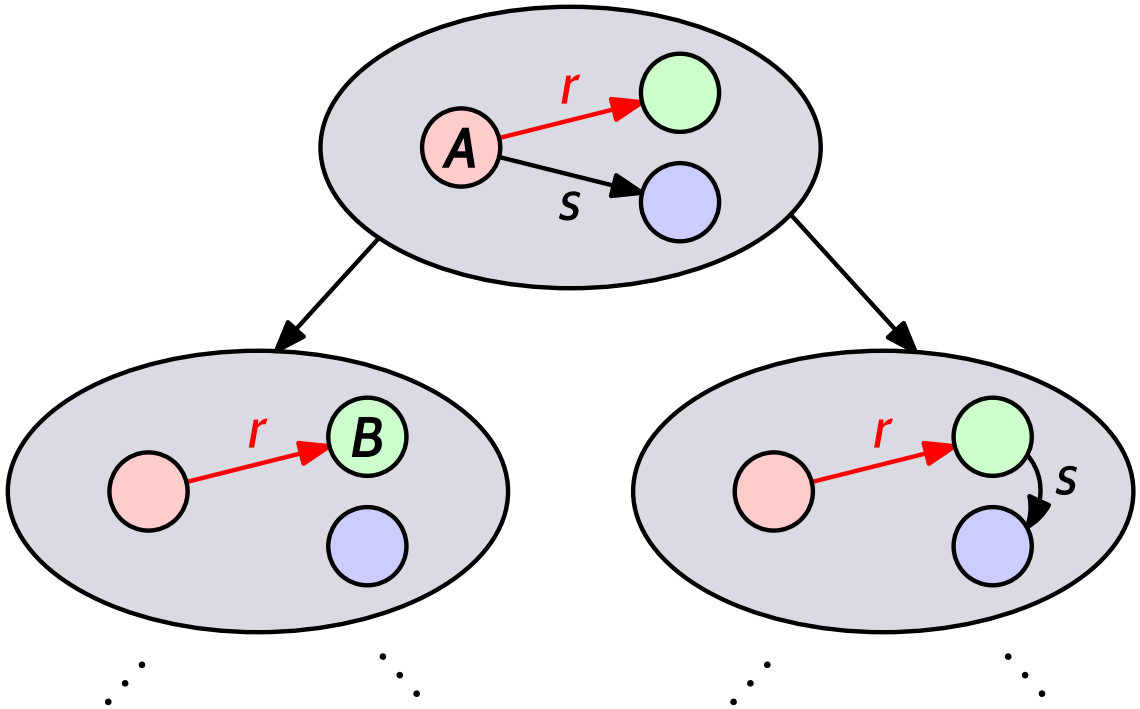
- Temporal operators from CTL
- Temporal concepts
- Acyclic TBoxes **(NEW)**

TDLs in a nutshell: semantics

Temporal dimension: worlds + tree-shaped 'future' relation

DL dimension: one full DL interpretation per world

- Constant domain assumption
- Rigid roles allowed



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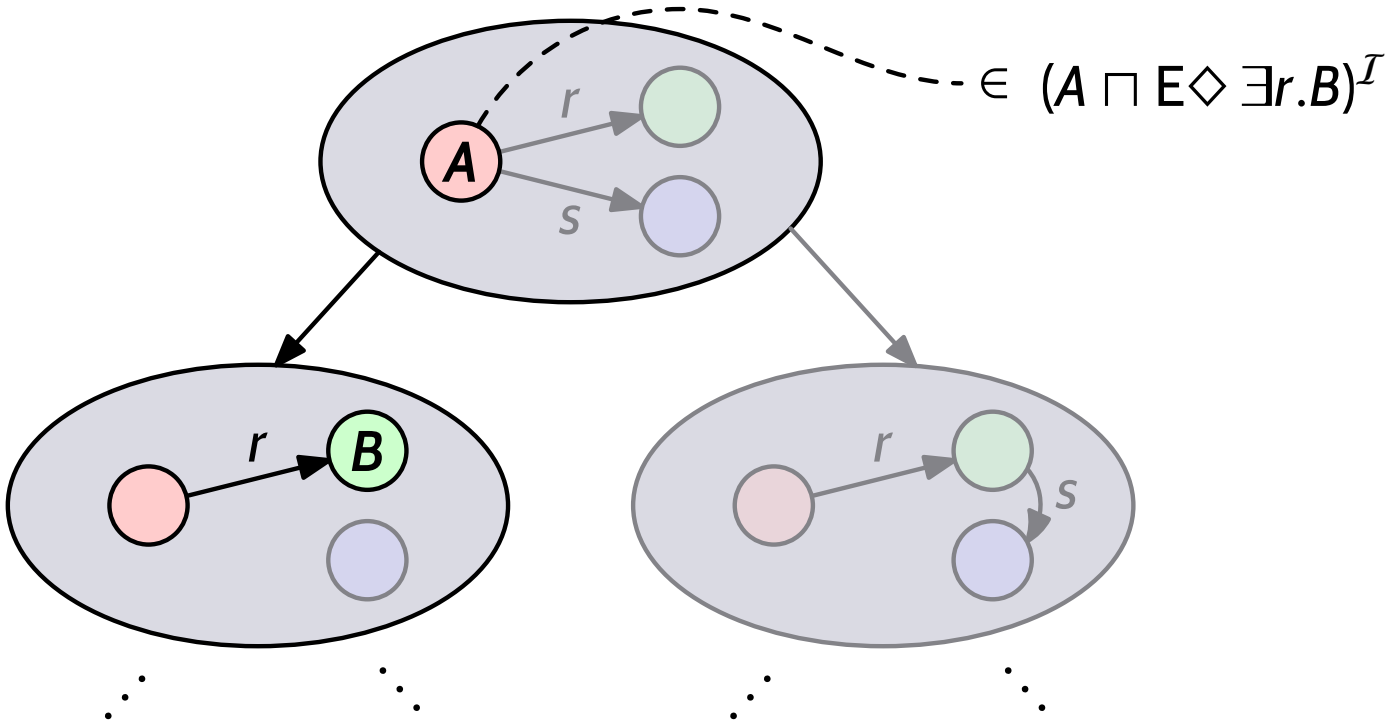

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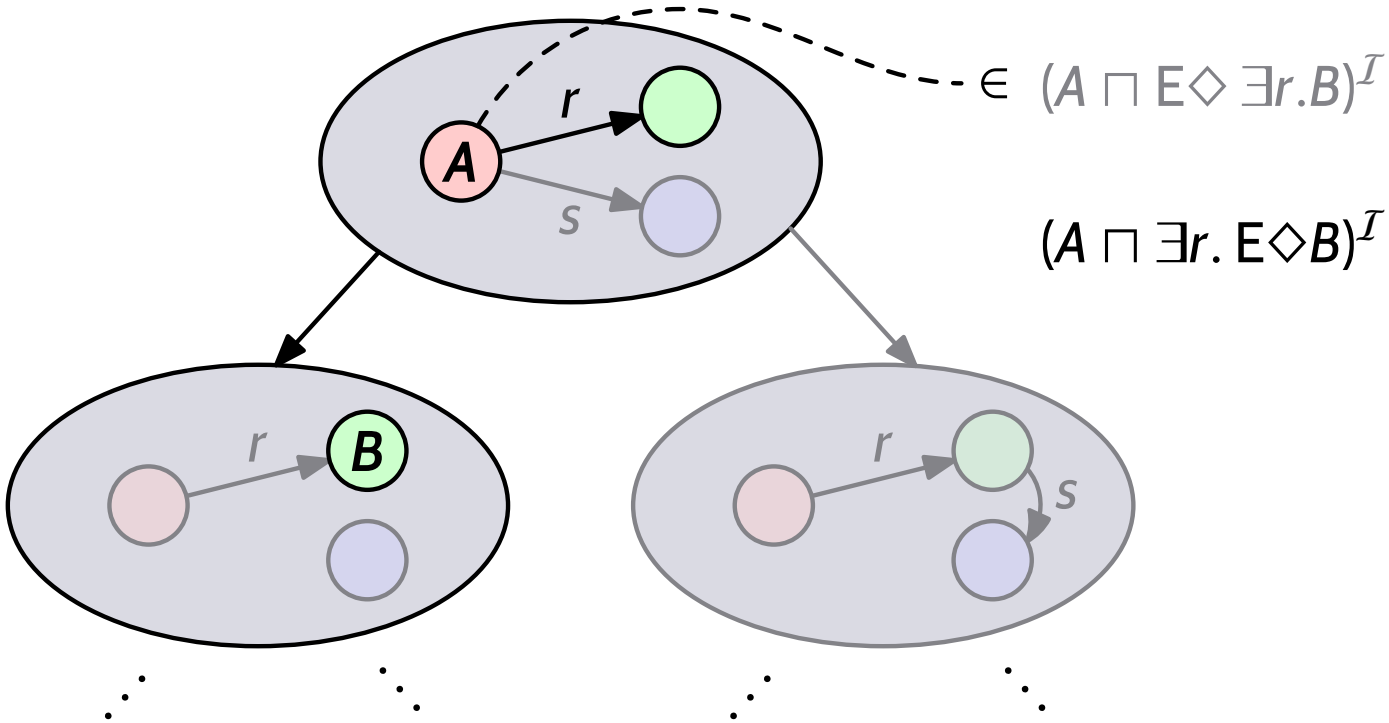

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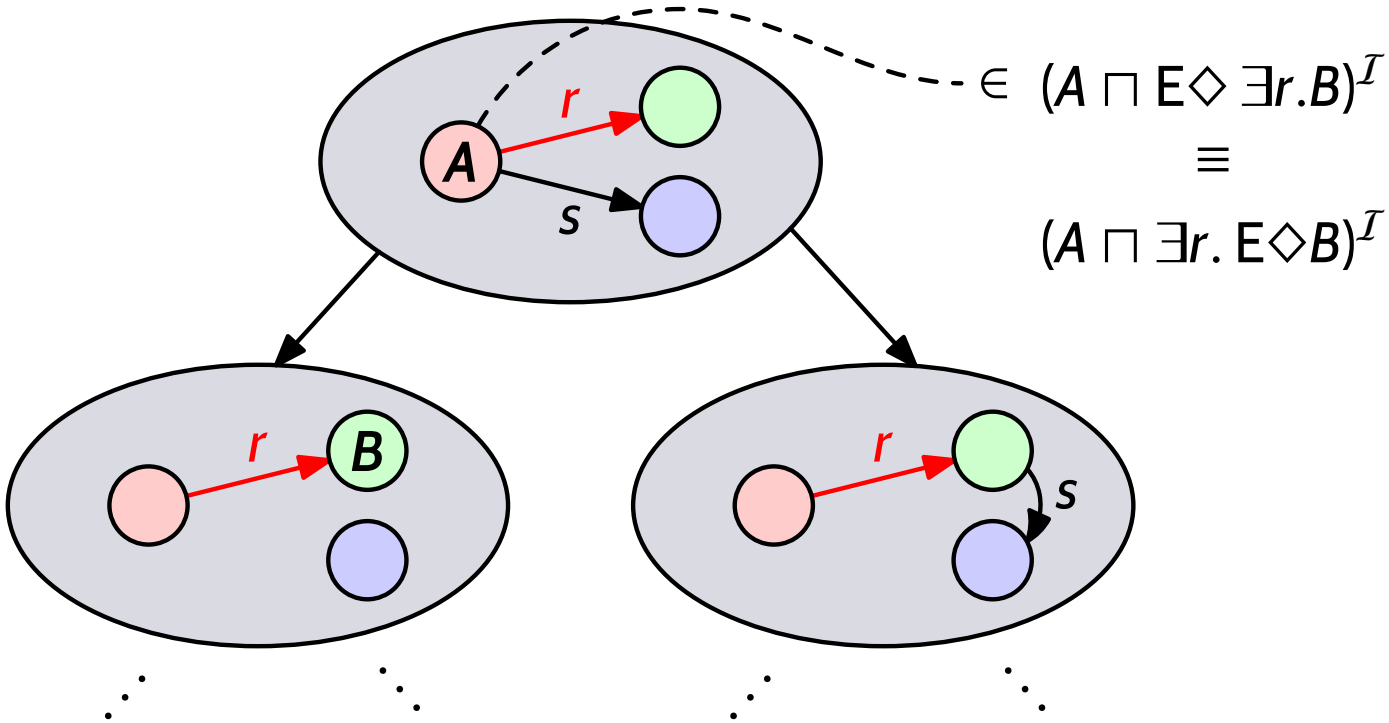

Contribution

We study decidability and complexity of subsumption

	Our results		[GJS KR'14]
$\mathcal{EL}$ + ...	empty TBox	acyclic TBoxes	general TBoxes
... E$\bigcirc$	in PTIME	**in PTIME**	undecidable
... E$\Diamond$	in PTIME	**in PTIME**	nonelementary
... E$\bigcirc$, E$\Diamond$	**coNP-complete**	in coNEXPTIME	undecidable
... E$\Diamond$, A$\Box$	in PSPACE	**PSPACE-complete**	undecidable
$\mathcal{ALC}$ + CTL	decidable but nonelementary		undecidable

- First fragments of $\mathcal{EL}$-based TDLs with rigid roles with elementary (even polynomial) complexity

The 2 main results (out of 4)

Theorem

1. $\mathcal{EL}$+E$\Diamond$ and $\mathcal{EL}$+E$\bigcirc$ over acyclic TBoxes are in PTIME.
2. $\mathcal{EL}$+{E$\Diamond$, A$\Box$} over acyclic TBoxes is PSPACE-complete.

Proof sketch

1. Build **abstract representation** of canonical model of input TBox, using 3-phase algorithm (thanks to acyclicity)
2. Upper bound:
   - abstract representation blows up $\rightsquigarrow$ consider single **traces**
   - complete the traces one at a time (think tableaux)
   - polynomial size bound thanks to acyclicity

   Lower bound: reduction from QBF

Conclusion

- ✓ Acyclic TBoxes can help design well-behaved $\mathcal{EL}$-based TDLs
- ✓ $\mathcal{EL}$ + CTL fragments of elementary (polynomial) complexity
- ✓ Byproduct: complexity results for
  positive fragments of product modal logics $\mathsf{K} \times \mathsf{K}$, $\mathsf{S4} \times \mathsf{K}$

- ? More expressive fragments
  e.g., $\mathcal{EL}$+{E$\bigcirc$, E$\Diamond$} (non-convex) over acyclic TBoxes
- ? Cyclic TBoxes
- ? Change the temporal component: LTL, μ-calculus?

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Thank you.