Integrating the Text-Editor $\text{T}_{\text{E}}\text{X}_{\text{MACS}}$ with the Proof Assistance System ΩMEGA using PLATΩ

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Research Goals

Background: Artificial Intelligence, Formal methods, deduction (especially, Inductive Theorem Proving)

- The potential of using formal logic based techniques is clearly recognized (mathematics, reliability of software systems, . . .)
- Formal logic based tools are not routinely employed like, for instance, CAS by mathematicians, physicists or engineers.
- Two reasons:
  - the high hurdle posed by the deep formal logical knowledge that is necessary to effectively use these techniques and tools
  - that the formal techniques and tools are not integrated with standard (software) development environments.

Past and Ongoing Research

Semantics-based management of change:

- Evolutionary nature of specification and verification: MAYA
  - Development graphs to represent and exploit theory structure (now semantics for theories and locales in Isabelle)
  - Difference Analysis & Propagation of changes to preserve proof knowledge

  with D. Hutter and T. Mossakowski

- Offspring: Semantics based XML difference computations (Masters, S. Radzevich)

  with D. Hutter

Past and Ongoing Research

Usability of formal reasoning tools:

- Assertion-Level (Huang'93) reasoning
  - (CoRe calculus, PhD 2003)
  - Tasklayer of the new version of the proof assistant ΩMEGA (Diploma thesis D. Dietrich)
- The next version of the ΩMEGA Proof assistant (Project in CRC "Ressource-adaptive cognitive processes (SFB-378)")
- Integrating reasoning systems as Web Services: MATHSERVE (PhD J. Zimmer)
- Automated reasoning in large structured theories: ATLAS EPSRC Visiting Researcher (with A. Bundy and R. McCasland)

Past and Ongoing Research

Augmenting authoring and development environments by formal techniques:

- Evolution of computer-supported mathematics (and hence to all areas that employ mathematics)
- Low user-friendliness of actual proof assistants
- Interfacing scientific text-editors instead of yet another proof assistant GUI (Project VeriMathDoc (with A. Fiedler) and Diploma thesis M. Wagner)

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Brings together research developments and experience from

- Evolutionary software development (MAYA),
- Semantics-based XML-DIFF,
- Proof construction at the assertion-level (CORE, Tasklayer)

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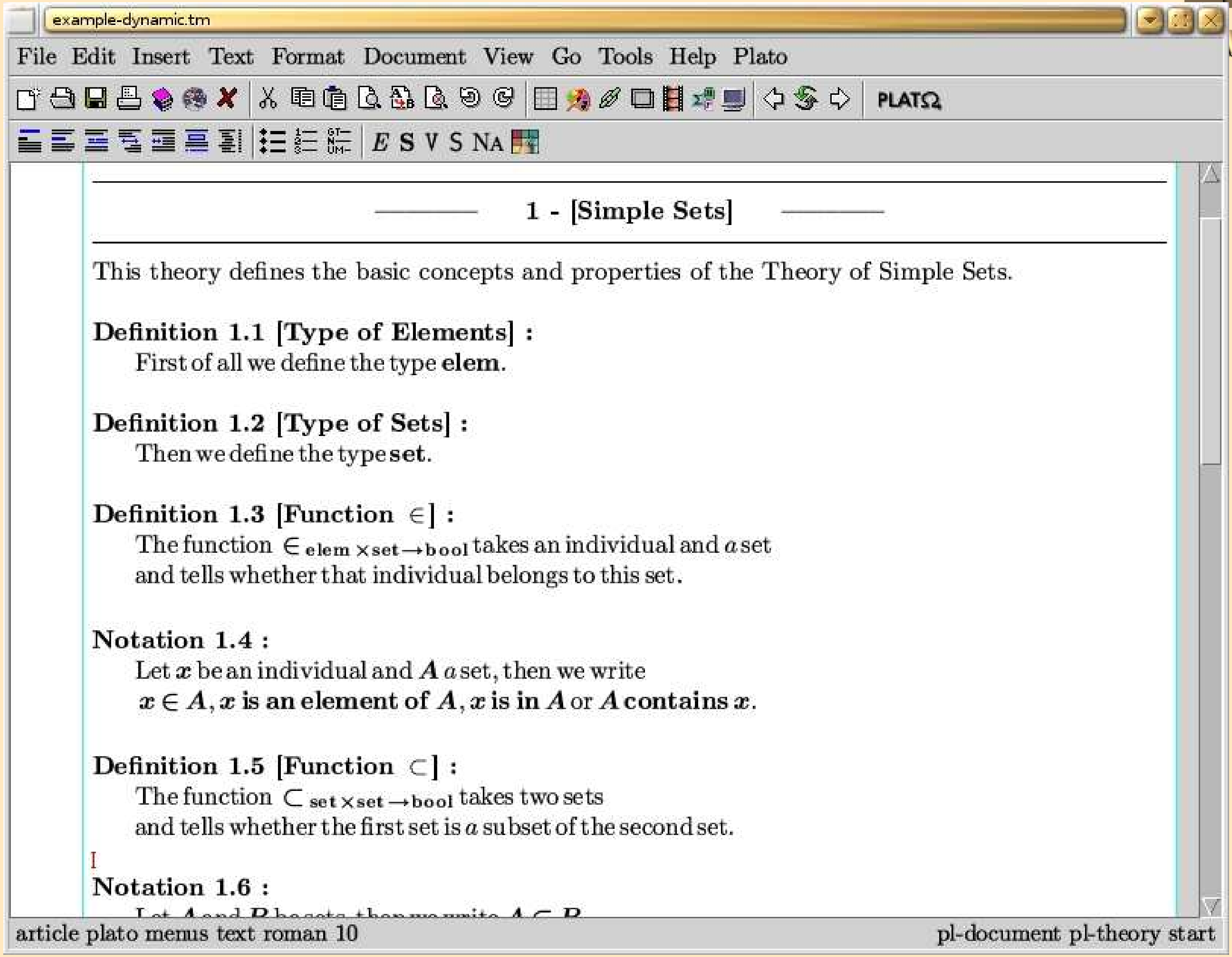


1 - [Simple Sets]

This theory defines the basic concepts and properties of the Theory of Simple Sets.

Definition 1.1 [Type of Elements] :
First of all we define the type **elem**.

Definition 1.2 [Type of Sets] :
Then we define the type **set**.

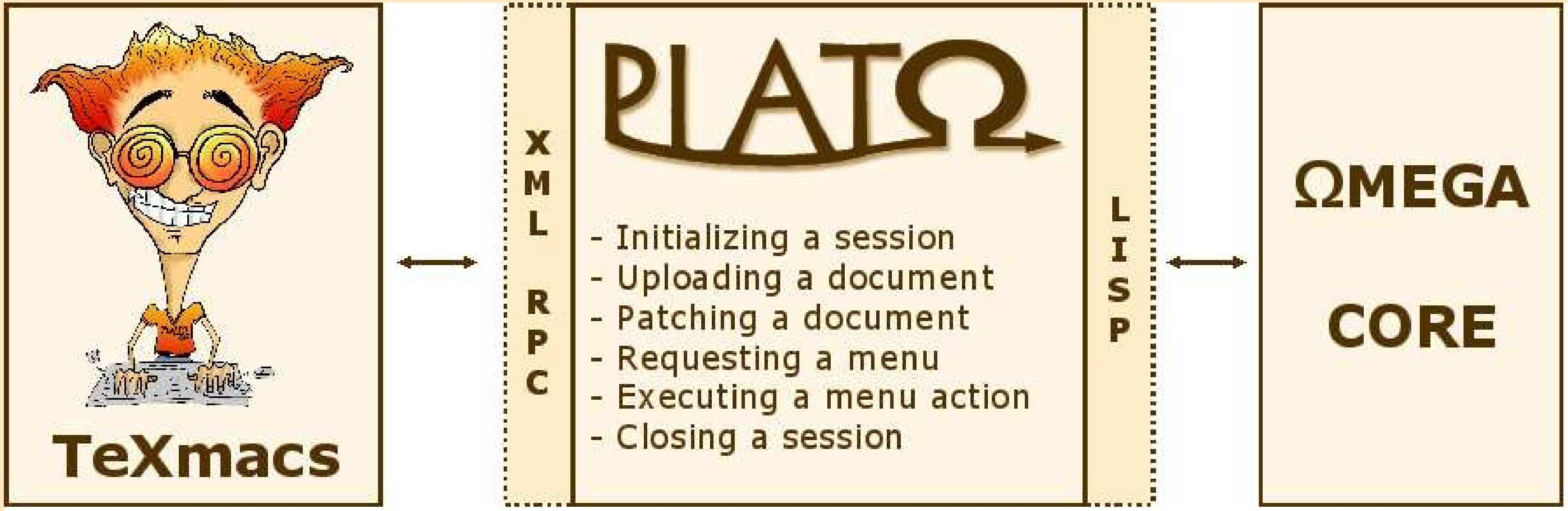
Definition 1.3 [Function $\in$] :
The function $\in_{\mathbf{elem} \times \mathbf{set} \rightarrow \mathbf{bool}}$ takes an individual and a set and tells whether that individual belongs to this set.

Notation 1.4 :
Let x be an individual and A a set, then we write
$x \in A$, x **is an element of** A, x **is in** A or A **contains** x.

Definition 1.5 [Function $\subset$] :
The function $\subset_{\mathbf{set} \times \mathbf{set} \rightarrow \mathbf{bool}}$ takes two sets and tells whether the first set is a subset of the second set.

Notation 1.6 :
Let A and B be sets, then we write $A \subset B$

System Architecture



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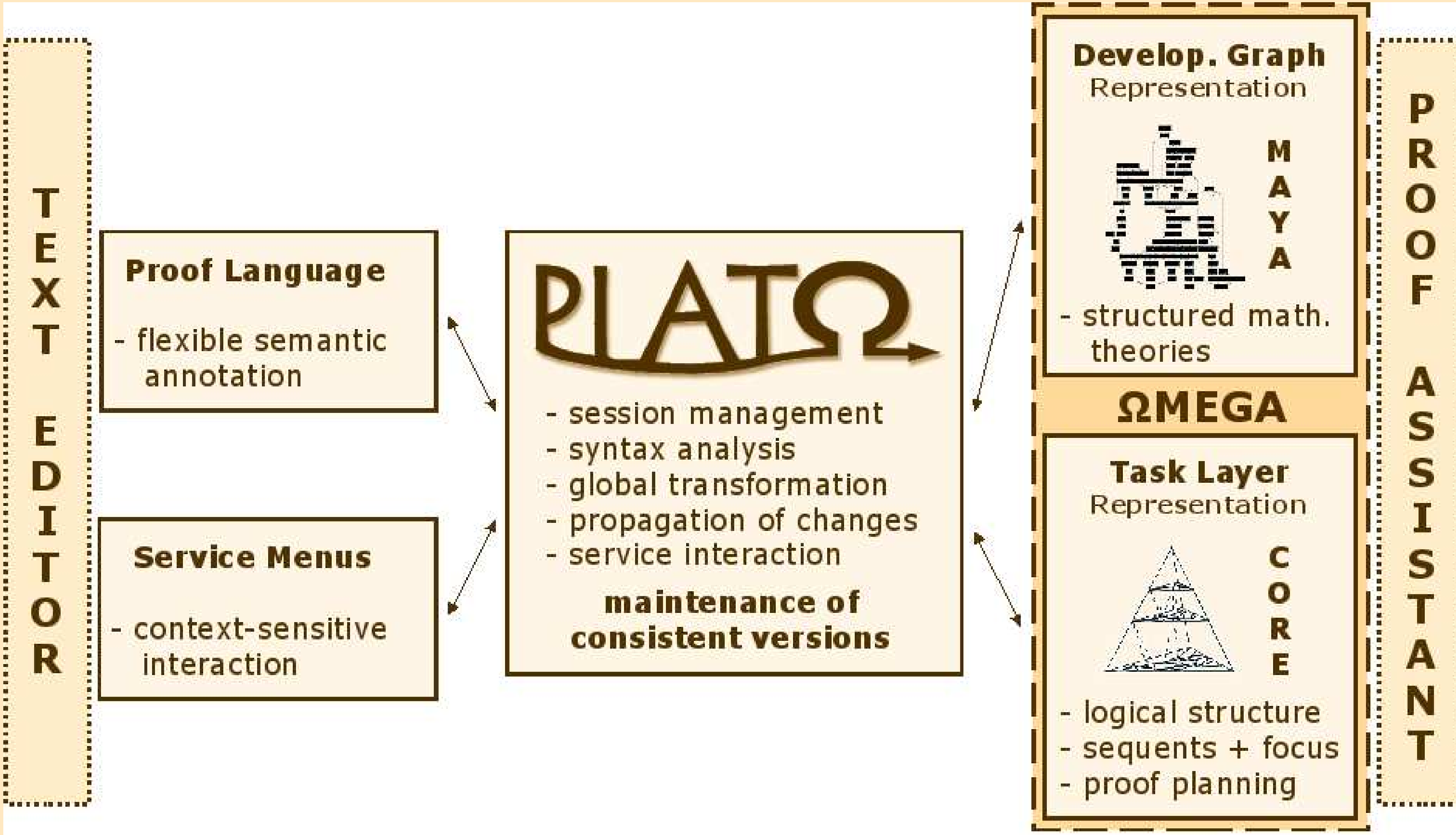
The Mediator PLATΩ

Mediation Problem



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Document Language

- **Semantic Annotation of Natural Language**
  - Macros in the text-editor
  - Individual layout style
- **Designed to support**
  - Textual structure of proofs
  - Flexible and multiple positioning
  - Underspecification
  - Alternative proof attempts
- **Parameters**
  - (Sub-)Languages for formulas, definitions and references

```
[document]
└─[theory]
   ├─[context]
   │  └─[reference]
   ├─[axiom]
   ├─[definition]
   ├─[theorem]
   │  ├─[precondition]
   │  │  └─[formula]
   │  └─[conclusion]
   │     └─[formula]
   └─[proof]
      ├─[cases]
      │  ├─[case]
      │  └─[proof]
      ├─[subgoals]
      ├─[assumption]
      ├─[fact]
      │  ├─[by]
      │  └─[obtain]
      └─[done]
```

Global Transformation

- Normalizing the flexible semantic annotations
- Separating proofs from theory knowledge
- Transforming linear proofs into the tree-like proof structure

Proof Language (PL)

- Theorems (flexible)
- Proofs (flexible, linear)
- Natural language text

Intermediate Language (IL)

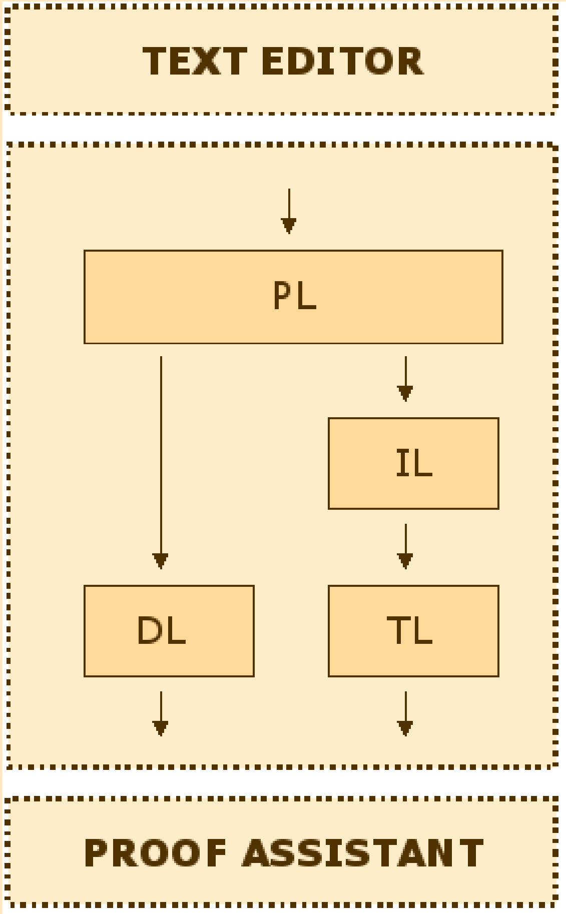
- Proofs (rigid, linear)

Development Graph Language (DL)

- Theorems (rigid)

Task Language (TL)

- Proofs (rigid, tree)



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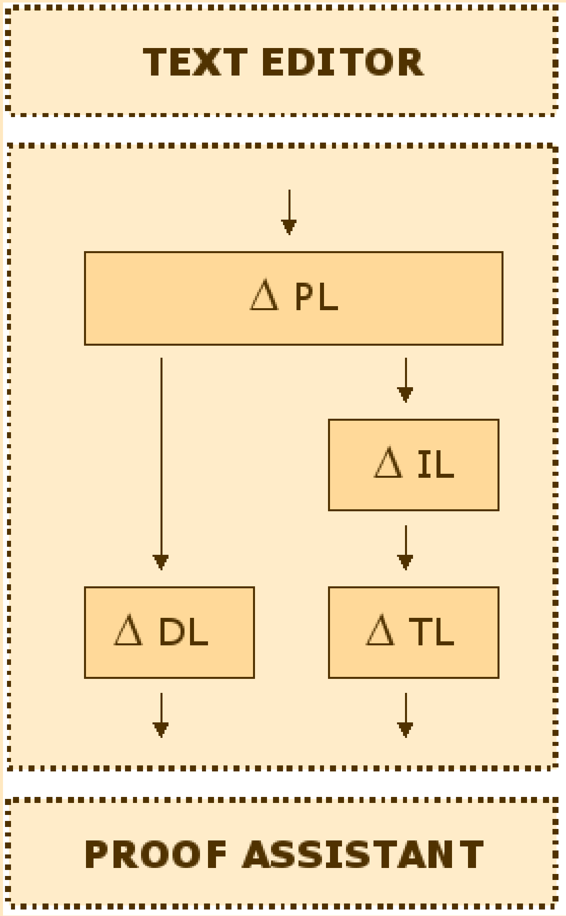
Propagation of Changes

- Semantic-based difference computation
- Efficient transformation of differences
- Preserve partial verifications in proof datastructure of PA



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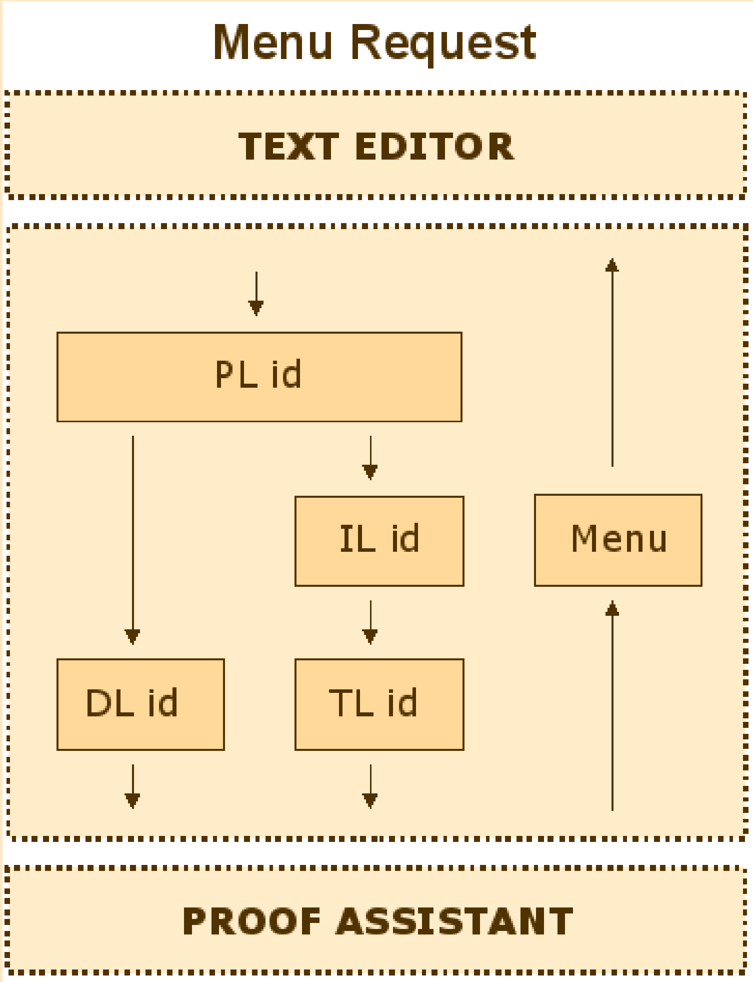
Task Language (TL)
- Proofs (rigid, tree)

Source: Autexier

Service Interaction: Requesting a Menu

- The user selects object in the text-editor
- Lookup all related objects in the maptable of the transformation
- Request a menu from the PA for these objects
- Display the menu in the text-editor



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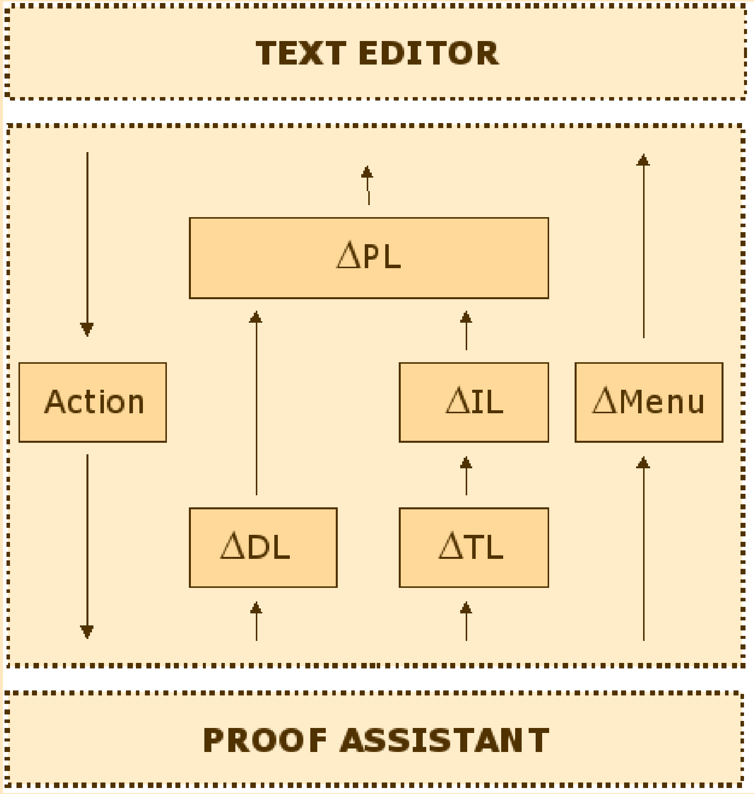
Service Interaction: Executing Actions

- User selects an action and its arguments in the menu
- Evaluate this action in the proof assistance system
- Evaluation result:
  - Patches to the menu differences (nested evaluation)
  - Patches to the DL and TL documents (top level evaluation)
- Transformation of DL/TL patches into PL patches



The Mediator PLATΩ

- XML-RPC server as interface for the text-editor
- Connection to proof assistant in Lisp
- Operating as online webservice or local plugin

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The Plugin in TEXMACS

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Rôle of the Plugin

- Key and session management
- Patch applications on the $\text{T}_{\text{E}}\text{X}_{\text{MACS}}$ document
- Resolve and check references

  $\boxed{\text{... by [Definition of } \subseteq \text{] ...}}$

- Communicates with PLATΩ by fully annotated documents
- Manually writing a fully annotated document is tedious
  - Have a context sensitive menu to write the annotations
  - May be acceptable for the user to write large structures (begin definition, theorem, proof, etc.)
  - But certainly not for formulas ...
    - To obtain: $\boxed{x \in U \Rightarrow x \in V}$
    - Write:

```
\imp{\in{\V{x}}{\V{U}}}{\in{\V{x}}{\V{V}}}
```

Writing Formulas

- **Use a parser for formulas**
- Allow the user to define its own syntax for any concepts

```
\notation{for=in}{
Let \declare{x} be an element and \declare{A} be a set.
Then we write \denote{x \in A}, \denote{x is in A},
\denote{x is element of A}, or \denote{A contains x}.}
```

Notation. Let **x** be an element and **A** be a set. Then we write **x** $\in$ **A**, **x is in A**, **x is element of A**, or **A contains x**.

- **Procedure:**
  - Scan all notation definitions, convert automatically into parser grammar rules
  - Using parser generator create (document) specific parser
  - Use that parser to parse formulas

The Parser Generator

- Standard LALR(1) parser generator implemented in Scheme
- Creates parser that generates all possible readings
- Allows specification of external call-backs to use to single-out invalid readings.
  Example: Could be used to integrate a “refiner”
- Sophisticated specification mechanism for precedences of operators

The Interface of the Proof Assistant

Source: Autexier

Features of the Proof Assistant

- Create inference rules from axioms/lemmas
  Example: $\forall A, B : set.(A \subseteq B \wedge B \subseteq A) \Rightarrow A = B$

  $$\frac{A \subseteq B \qquad B \subseteq A}{A = B}$$

  Mixture of CoRe ideas and B. Wacks' super natural deduction
- Proof construction on that level (assertion level)
- Requires proof representation that allows encoding of proof continuations
- Every (additional) feature of the PA is immediately available on the corresponding text part.
  Example: Automatic Theory Exploration System MathsAid

Source: Autexier

System Demo

- Definition of concepts and their notations
- Writing Axioms, Conjectures using pre-defined and user-defined notations
- Proof support: interactive and automatic

Source: Autexier

Related Work

- **Automath, Mizar, Isar:**
  - Balanced compromise between machine processable and human readable
- **Grammatical Framework:**
  - Framework to define grammar for an abstract and a concrete syntax
- **PCOQ:**
  - Schematic quasi-natural language output
- **Nuprl, Clam, Omega/P.Rex:**
  - Natural language processsing technique to generate proof descriptions

Related Work

- **Theorema:**
  - Strictly separated formal and informal parts
- **Mathlang:**
  - Top-down from natural language
  - Use annotations for structure, no parser as well
  - Still even more far away from PAs
- **ProofGeneral:**
  - Top-down processing of documents
  - Documents are input format of PA rather than of some typesetting program.

Conclusion

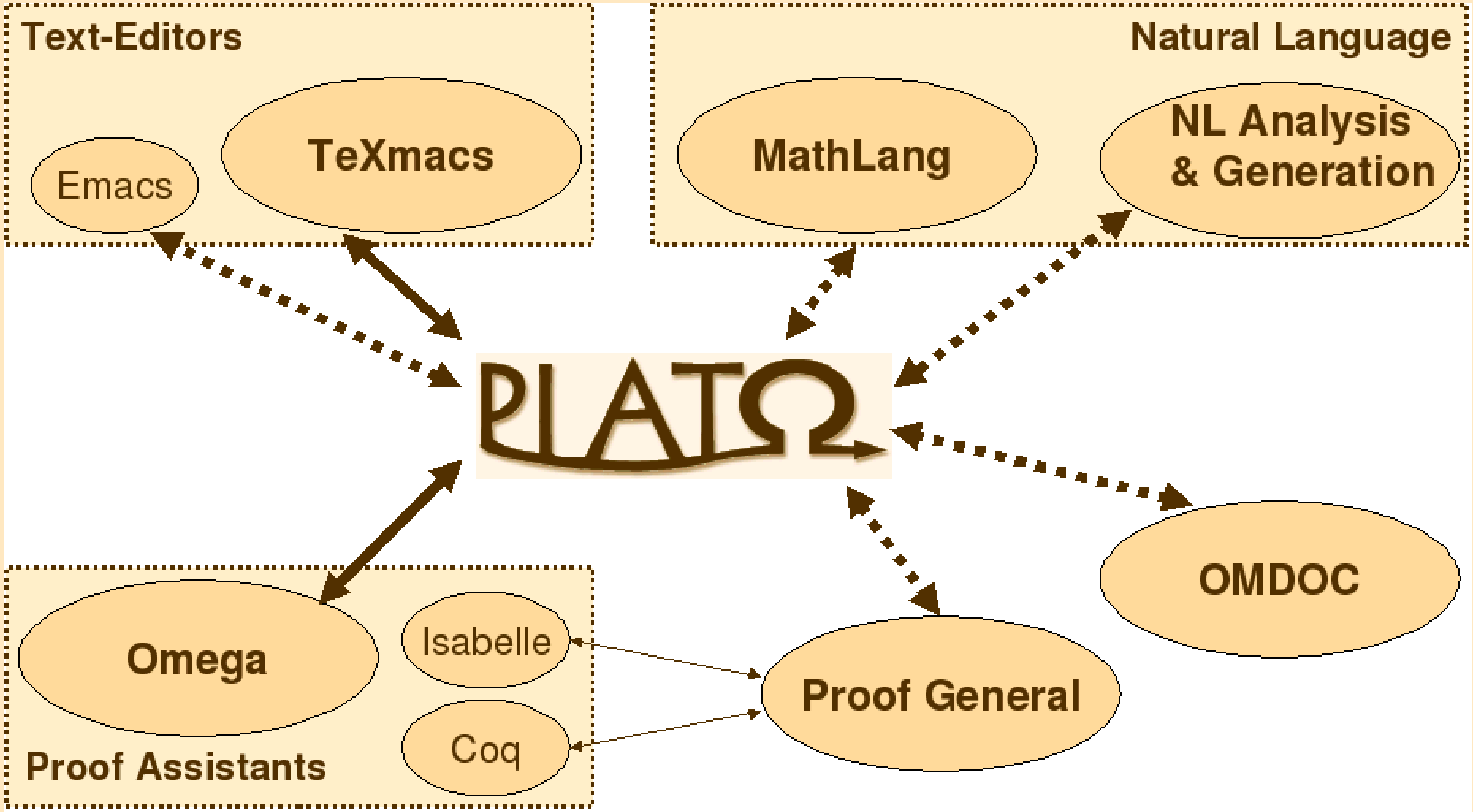
- **Have a stable connection between $\text{T}_\text{E}\text{X}_{\text{MACS}}$ and ΩMEGA**
  - PLATΩ deals with all mediating (translation, consistency, patching, relationship between parts of text and formal parts in PA, menus)
  - Clean interface to text-editor
  - Parameterized over language for formulas, definitions and references
- **Either side can be enhanced without affecting the mediator**
  - PLATΩ plugin:
    - User-definable notation used when parsing formulas
    - Add more NL analysis to automatically annotate text(parts)
    - Add NL generation (incremental)
  - ΩMEGA:
    - Added theory exploration, classical ATPs are available
    - Increase proof search automation

Source: Autexier

Future Work

- **Technicalities:**
  - XML-RPC Interface for PAs (OMDOC based)
  - Asynchronous communications
- **Editing:**
  - Library mechanism
  - Dependent types (Scunak, ΩMEGA'07, others are welcome)
  - Support overloading
  - More natural language analysis and NL generation
  - Semantics-based versioning, collaborative editing
- **Proof support**
  - Automation of proof search at the assertion-level
  - Exploit structure of and in theories to enhance automatic proof search

Future Work


Text-Editors
Emacs
TeXmacs
Natural Language
MathLang
NL Analysis
& Generation
PLATΩ
OMDOC
Omega
Isabelle
Coq
Proof General
Proof Assistants